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Reg. No. : ....

Name : ....

Second Semester B.Sc. Degree Examination, May 2020

Career Related First Degree Programme Under CBCSS

Group 2 (a) – Complementary Course II for Physics and Computer Applications

MM 1231.6 – MATHEMATICS – II PARTIAL DIFFERENTIATION, VECTOR DIFFERENTIATION, COMPLEX NUMBERS AND MULTIPLE INTEGRALS

(2019 Admission)

Time : 3 Hours Max. Marks : 80

SECTION – I

All the first ten questions are compulsory. They carry 1 mark each.

1. Find $\frac{\partial^2 f}{\partial x\, \partial y}$ of the function $f(x, y) = x^2 + y^2 + 4$.

2. Determine whether $(3x + 2)\, y\, dx + x(x + 1)\, dy$ in exact or not.

3. Define gradient of a scalar field.

4. Find the divergence of the vector field.

   $\bar{a} = 2xz^2\, \hat{i} - yz\hat{j} + 3xz^3\hat{k}$ at $(1, 1, 1)$.

5. Find the Laplacian of the scalar field $\phi = x^2y - 3y^2z^2$.

6. State de Moivres theorem.

7. Find $\sinh(ix)$.

P.T.O.

8. Write the real part of e^z.

9. Write the Jacobian of the co-ordinate transformation $x = g(u,v)$, $y = h(u,v)$.

10. Evaluate $\int_0^2 \int_0^{2-x} dy\, dx$.

SECTION – II

Answer **any eight** from among the questions 11 to 22. These questions carry **2** marks each.

11. Find the total derivative of the function $f(x,y) = y\exp(x+y)$.

12. Using chain rule find $\frac{df}{dt}$ given $f(x,y) = 4x^2 + 3y^2$, $x(t) = \sin t$, $y(t) = \cos t$.

13. State Taylor's theorem for two variables x and y.

14. Determine the stationary point of the function

    $f(x,y) = xy - x^2 - y^2 - 2x - 2y + 4$ and describe the nature of the function at those points.

15. The position vector of a particle at time t is Cartesian coordinate is given by

    $r(t) = 2t^2\hat{i} + (t^2 - 4t)\,\hat{j} + (3 + -5)\,\hat{k}$. Find at time $t = 1$ the speed of the particle and component of velocity in the direction $\bar{s} = \hat{i} - 3\hat{j} + 2\hat{k}$.

16. Find the curl of the vector field $\bar{a} = xyz\,\hat{i} + 3x^2\hat{j} + (xz^2 - y^2z)\,\hat{k}$ at $(1, 2, -1)$.

17. Prove $curl(grad\ \phi) = 0$.

18. Find the solution of the equation $z^3 = 1$.

19. Express $\cos 3\theta$ and $\sin 3\theta$ in terms of powers of $\cos\theta$ and $\sin\theta$.

20. Evaluate $L_n(-i)$.

21. Find the Jacobian $\dfrac{d(x, y, z)}{\partial(g, v, w)}$ of the transformation $x = u\cos v$, $y = u\sin v$, $z = w$.

22. Evaluate $\int_0^3 \int_0^{\sqrt{9-x^2}} \int_0^{\sqrt{9-x^2}} dz\, dy\, dx$.

SECTION – III

Answer **any six** questions from among the questions 23 to 31. These questions carry **4** marks each.

23. The temperature of a point (x, y) on a unit circle is given by $T(x, y) = 1 + xy$. By using the method of Lagrange multipliers, find the temperature of the hottest point on the circle.

24. Find the total derivative of the function $f(x, y) = xy^2 + x^2y$ with respect to x given $y = \ln x$.

25. Find the Taylor expansion, up to quadratic terms in x and y of $f(x, y) = \sin 2x + \cos y$ about the point $(0, 0)$.

26. Find the directional derivative of the function $\phi(x, y, z) = 2xy + z^2$ is the direction of the vector $\bar{a} = \hat{i} + 2\hat{j} + 2\hat{k}$ at the point $(1, -1, 3)$.

27. For $\bar{r} = x\hat{i} + y\hat{j} + z\hat{k}$ and $|\bar{r}| = r$. Show that $\nabla f(r) = \dfrac{1}{r} f'(r)\, \bar{r}$.

28. Solve the hyperbolic equation $\cosh x - 5\sinh x - 5 = 0$.

29. Find the closed from expression for the inverse hyperbolic functions $y = \sinh^{-1} x$.

30. Evaluate the double integral $I = \iint_R x^2 y \, dx \, dy$ where R is the triangular area bounded by the lines $x = 0$, $y = 0$ and $x + y = 1$. Also reverse the order of integratioh and show that same result is obtained.

31. Find the expression for the volume element is spherical polar coordinate.

SECTION – IV

Answer **any two** questions from among the questions 32 to 35. These questions carry **15** marks each.

32. (a) Show that the function $f(x, y) = x^3 \exp(-x^2 - y^2)$ has a maximum at the point $\left(\frac{\sqrt{3}}{2}, 0\right)$ a minimum at $\left(-\frac{\sqrt{3}}{2}, 0\right)$ and a stationary point at the origin.

(b) Find the stationary point of $f(x, y, z) = x^2 - 2y - z^2$ subject to constraints $g(x, y, z) = 2x - y = 0$ and $h(x, y, z) = y + z = 0$.

33. Show that $(\nabla\phi \times \nabla\psi)$.

(a) $\nabla. = 0$ where ϕ and ψ are scalar fields.

(b) $\nabla \times (\phi \, \bar{a}) = \nabla\phi \times \bar{a} + \phi \, \nabla \times \bar{a}$ where ϕ scalar field.

34. (a) Draw the graph of $\cosh x$, $\sec x$.

(b) Use de Moivres theorem to prove that $\cos 4\theta = \cos^4\theta - 6\cos^2\theta \sin^2\theta + \sin^4\theta$.

35. (a) Find the volume integral of $x^2 y$ over the tetrahedral volume bounded by the planes $x = 0$, $y = 0$, $z = 0$ and $x + y + z = 1$.

(b) Find the area of the region R enclosed by the parabola $y = x^2$ and the line $y = x + 2$.
