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Reg. No. : ....

Name : ....

Second Semester B.Sc. Degree Examination, August 2018
First Degree Programme under CBCSS
Complementary Course for Physics
MM 1231.1 : MATHEMATICS II : Integration and Vectors
(2014 Admission Onwards)

Time : 3 Hours Max. Marks : 80

SECTION – I

All the **first 10** questions are **compulsory**. They carry **1** mark **each**.

1. Suppose that a particle moves along a coordinate line so that its velocity at time t is v (t) = sint. Find the displacement of the particle during the time interval, $0 \leq t \leq \frac{\pi}{2}$.
2. Find the average value of $f(x) = -3x^2 - 1$ on [0, 1].
3. Evaluate $\int \tan^2 x \, dx$.
4. Evaluate $\int_0^1 \int_0^2 (x+3)\, dy dx$.
5. Find the unit tangent vector at a point t to the curve $\vec{r} = a\cos t\,\vec{i} + a\sin t\,\vec{j}$.
6. A particle moves so that its position vector is given by $\vec{r}(t) = a\cos\omega t\;\vec{i} + a\sin\omega t\;\vec{j}$, where ω is a constant. Show that the velocity of the particle is perpendicular to $\vec{r}$.
7. Find div $\vec{F}$, if $\vec{F}(x,y,z) = x^2\vec{i} - 2\vec{j} + yz$.
8. Show that the field $\vec{F} = yz\vec{i} + xz\vec{j} + xy\vec{k}$ is conservative.
9. Find the area of the region enclosed by ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ by using Greens theorem.
10. If f (x, y, z) = $\sin x + e^{xy} + z$, find ∇f.

P.T.O.

SECTION – II

Answer **any 8** questions from among the questions **11** to **22**. These question carry **2** marks **each**.

11. A ball is hit directly upward with an initial velocity of 58 m/sec and is stuck at a point that is 1 m above the ground. Assuming that the free fall model applies, how high will the ball travel ?

12. Find the area of the region bounded above by $y = x + 4$ and the lower boundary is $y = x^2$ and bounded on the side by $x = 2$ and $x = 4$.

13. Evaluate $\int x^2 \sec^2 (x^3)\, dx$.

14. Find the volume of solid generated by revolving the region between y-axis and the curve $x = \frac{2}{y}$, $2 \leq y \leq 4$ about the y-axis.

15. Evaluate $\iint xy\, dxdy$ over the region R in the first quadrant of the circle $x^2+y^2 = a^2$.

16. Find the slope of the line in 2 space that is represented by the vector equation $\vec{r} = (6 - 3t)\vec{i} - (2 - 5t)\vec{j}$.

17. Show that $\nabla^2 \left(\frac{1}{r}\right) = 0$, where $r = \sqrt{x^2 + y^2 + z^2}$.

18. Find the values a, b, c so that $\vec{v} = (x + 2y + az)\vec{i} + (bx - 3y - z)\vec{j} + (4x + cy + 2z)\vec{k}$ is irrotational.

19. Find the directional derivative of $f(x, y, z) = x^3 - xy^2 - z$ at (1, 1, 0) in the direction of $2\vec{i} - 3\vec{j} + 6\vec{k}$.

20. Find the work done by a force $\vec{F} = xy\vec{i} + y\vec{j} - yz\vec{k}$ over the curve $\vec{r}(t) = t\vec{i} + t^2\vec{j} + t\vec{k},\ 0 \le t \le 1$.

21. Evaluate outward flux of the vector field $\vec{F}(x, y, z) = 7x\vec{i} - z\vec{k}$ across the sphere $\sigma : x^2 + y^2 + z^2 = 9$ by using divergence theorem.

22. Evaluate $\int (x + y)\, ds$ over the straight line segment $x = t$, $y = (1 - t)$, $z = 0$ from (0, 1, 0) to (1, 0, 0).

SECTION – III

Answer **any 6** questions from among the questions **23** to **31**. These questions carry **4** marks **each**.

23. Find the length of the curve $x = a\cos^3\theta$, $y = a\sin^3\theta$.

24. Change the order of integration and hence evaluate the double integral $\int_0^1 \int_{e^x}^{e} \frac{dxdy}{\log y}$.

25. Find the area enclosed by the lemniscates $r^2 = 4\cos 2\theta$.

26. Using the idea of triple integral, find the volume of the solid enclosed by the sphere $x^2 + y^2 + z^2 = 4$.

27. Find the radius of curvature at a point t for the helix $\vec{r}(t) = a\cos t\vec{i} + a\sin t\vec{j} + bt\vec{k},\ a \ge 0,\ b \ge 0$.

28. Prove that curl $(\text{curl}\vec{F}) = \text{grad div } \vec{F} - \nabla^2 F$.

29. Show that the integral $\int_{(-1, 5)}^{(4,3)} 3z^2 dx + 6xz dz$ is independent of the path and hence evaluate the integral.

30. Find :

   a) The unit vector normal to the surface $f(x, y, z) = x^2 + y^2 - z$ at the point (2, –2, 3).

   b) Maximum possible $\frac{df}{ds}$ at the point (1, 4, 2) of the surface $f(x, y, z) = x^2 + y^2 - z$.

31. Use spherical coordinates to evaluate $\int_{-2}^{2} \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \int_{0}^{\sqrt{4-x^2-y^2}} z^2 \sqrt{x^2 + y^2 + z^2}\, dzdydx$.

SECTION – IV

Answer **any 2** questions from among the questions **32** to **35**. These questions carries **15** marks **each**.

32. Find :

a) The area of the surface generated by revolving right hand loop of the lemniscate $r^2 = \cos 2\theta$ about the y-axis.

b) Find the volume of the region D enclosed by the surfaces $z = x^2 + 3y^2$ and $z = 8 - x^2 - y^2$.

33. a) Show that $\iint F \cdot n\, ds = \frac{12}{5}\Pi r^5$, where surface S is a sphere center origin and radius r and $\vec{F} = x^3\vec{i} + y^3\vec{j} + z^3\vec{k}$.

b) Show that $\iint r.n\, ds = 3V$, where V is the volume enclosed by the surfaces S and $\vec{r}$ is the position vector.

34. a) By using Greens theorem find the work done by the force field $\vec{F} = (x^2 + y^2)i - 2xyj$ on a particle that travels once around the rectangle in the xy plane bounded by x = 0, x = a y = 0, y = b.

b) Show that divergence of inverse square field $\vec{F}(r) = \frac{c}{\|r\|^3}\vec{r}$ is zero.

35. a) Find the volume of the upper region D cut from the solid sphere $\rho \le 1$ by the cone $\varphi = \frac{\pi}{3}$.

b) Find the area of the surface generated by revolving the arc of the catenary $y = c\cosh\frac{x}{c}$ from x = 0 to x = c about the x-axis.