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Reg. No. : ....

Name : ....

Second Semester B.Sc. Degree Examination, December 2021

First Degree Programme Under CBCSS

Mathematics

Complementary Course for Physics

MM 1231.1 MATHEMATICS II — CALCULUS WITH APPLICATIONS IN PHYSICS – II

(2018 and 2019 Admission)

Time : 3 Hours Max. Marks : 80

SECTION – I

All 10 questions are compulsory, each carries **1** mark.

1. Find the modulus of $z = a - ib$.

2. Find the principal arguments of $z = 4i$.

3. Find all second order partial derivatives of $f(x, y) = 2x^3y^2 + y^3$.

4. Find total derivatives of $z = ye^{x+y}$.

5. Show that the differential form $(x^2 + 2x)dx + y^2dy$ is exact.

6. Show that $(y + z)dx + xdy + xdz$ is an exact differential form.

7. Find $\int_0^a\int_0^b\int_0^c dxdydz$.

P.T.O.

8. If $\vec{r}(t) = 5t^2\hat{i} + (t+2)\hat{j} = 3t^3\hat{k}$ then $\frac{d\vec{r}}{dt}$.

9. If $F(x, y, z) = (x + y)\hat{i} + (y + z)\hat{j} + (x + z)\hat{k}$. Find $DivF$ at (1, 1, 2).

10. If $\varphi(x, y, z) = x + y + z$, find $grad\varphi$.

SECTION – II

Answer any **eight** questions, each carries **2** marks.

11. Verify the result $|z_1 z_2| = |z_1||z_2|$ if $z_1 = 1 + i$ and $z_2 = \frac{2}{i}$.

12. If $z = a + ib$ then show that $zz^* = |z|^2$.

13. If $z = re^{i\theta}$ then find the value of $z^n - \frac{1}{z^n}$.

14. Show that $\cosh ix - \cos x$.

15. Show that $\cosh^2 x - \sinh^2 x = 1$.

16. Find the total derivative of $f(x, y) = x^2 + 3xy$ with respect to x, given $y = \sin^{-1} x$.

17. Find the local extreme values of $f(x, y) = xy$.

18. Suppose $f(x.y) = xy$ defined over R in xy-plane given by, $0 \le x$, $y \le 1$, then find average of f over the region R.

19. $\int_0^1 \int_0^x (x + y)dydx$.

20. If $\varphi(x, y, z) = x^2 + y^2 + z^2$ then find $grad\varphi$ at (1,2,3).

21. Find Laplacian of $\varphi(x, y, z) = e^x \sin y + z$.

22. Show that $F(x, y, z) = (x + y)\hat{i} + (x - y)\hat{j} + (x + z)\hat{k}$ is solenoidal.

SECTION – III

Answer any **six** questions. **Each** question carries **4** marks.

23. Find the derivative of $e^{3x}\cos 4x$ with respect x using complex exponential form

24. Find the principal values of (a) $z = \ln(-i)$ (b) $z = i^{-2i}$.

25. Find a closed form expression for the inverse hyperbolic function $y = \sinh^{-1} x$.

26. Find the Taylor expansion up to quadratic terms in $(x - 2)$ and $(y - 3)$ of $f(x, y) = ye^{xy}$ about the point $x = 2$ *and* $y = 3$.

27. The temperature of a point (x, y) on a unit circle is given by $T(x, y) = 1 + xy$. Find the temperature of the two hottest point on the circle.

28. Find the area bounded by the curves $y = x^2$ above x *axis* and below the line $y = 2$ using the concept of double integral.

29. If $x = r\cos\theta$ and $y = r\sin\theta$ then show that $\frac{\partial(x,y)}{\partial(r,\theta)} \cdot \frac{\partial(r,\theta)}{\partial(x,y)} = 1$.

30. Show that $\nabla \cdot (\nabla\varphi) = \nabla^2\varphi$ for any scalar field φ and verify for $\varphi(x, y, z) = x^2 + y^2 + z^2$.

31. If $F(y, z) = z^2(x^2y^2\hat{i} + y^2\hat{j} + x^2\hat{k})$ then find $\nabla \cdot (\nabla \times F)$.

SECTION – IV

Answer any **two** questions, each carries **15** marks.

32. (a) Solve the equation $z^6 - z^5 + 4z^4 - 6z^3 + 2z^2 - 8z + 8 = 0$.

(b) Using complex exponential form find $\int e^{ax}\sin bx dx$ and $\int e^{ax}\cos bx dx$.

33. Find the stationary points of $f(x, y, z) = x^3 + y^3 + z^3$ subject to the following conditions

(a) $g(x, y, z) = x^2 + y^2 + z^2 = 1$.

(b) $g(x, y, z) = x^2 + y^2 + z^2 = 1$ and $h(x, y, z) = x + y + z = 0$.

34\. (a) Find and expression for a volume element in spherical polar coordinates and hence calculate the moment of inertia about the diameter of a uniform sphere of radius '*a*'.

(b) Find $\int_0^4 \int_{x=\frac{y}{2}}^{\frac{y}{2}+1} \frac{2x-y}{2}\,dxdy$ by applying the transformation $u = \frac{2x-y}{2}$ and $v = \frac{y}{2}$.

35\. (a) Find the unit tangent vector $\hat{t}$ and acceleration $\vec{a}$ of a particle travelling along the trajectory given $\vec{r}(t) = 5\cos t\hat{i} + 5\sin t\hat{j} + 3t\hat{k}$.

(b) Show that the acceleration of a particle travelling along a trajectory $\vec{r}(t)$ is given by $\vec{a}(t) = \frac{dv}{dt}\hat{t} + \frac{v^2}{\rho}\hat{n}$, where $\hat{t}$ is the unit tangent vector, v is the speed, $\hat{n}$ is the principal normal vector and ρ is the radius of convergence.