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H – 2242

Reg. No. : ....

Name : ....

First Semester B.Sc. Degree Examination, November 2019

Career Related First Degree Programme under CBCSS

Complementary Course I for Physics and Computer Applications

MM 1131.6 : Mathematics I — CALCULUS, INFINITE SERIES AND VECTOR ALGEBRA

(2019 Admission)

Time : 3 Hours Max. Marks : 80

SECTION – I

All the first **ten** questions are compulsory. They carry **1** mark each :

1. State chain rule of differentiation.

2. Find the derivative of $= a^x$.

3. State Rolle's Theorem.

4. State rule of integration by parts.

5. What is the mean value of a function $f(x)$ between $x = a$ and $x = b$?

6. Find the sum to infinity of a geometric series having first term $\frac{1}{2}$ and common ratio $\frac{1}{2}$.

7. Sum the series $S = 2 + \frac{5}{2} + \frac{8}{2^2} + \frac{11}{2^3} + \ldots$

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8. State D'Alembert's ratio test.

9. What is vector triple product?

10. Define reciprocal vector.

(10 × 1 = 10 Marks)

SECTION – II

Answer **any eight** questions from among the questions 11 to 22. These questions carry **2** marks each.

11. Find the derivative with respect to x of $x^3 \sin x$.

12. Differentiate $\frac{\sin x}{x}$.

13. What are the three types of stationary points?

14. Evaluate $\int x^2 e^{-x^2} dx$.

15. Evaluate $\int_0^2 (2-x)^{\frac{-1}{4}} dx$.

16. Find the volume of a cone enclosed by the surface formed by rotating about x-axis the line $y = 2x$ between $x = 0$ and $x = b$.

17. Evaluate the sum $\sum_{n=1}^{2} \frac{1}{n(n+2)}$.

18. Test for convergence the series $\sum_{n=1}^{\infty} \frac{1}{n!+1}$.

19. Determine whether the series $\sum_{n=1}^{\infty} \left(\frac{1}{n}\right)^n$ is convergent.

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20. Find $\mathbf{a} \cdot \mathbf{b}$, where $\mathbf{a} = \mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$ and $\mathbf{b} = 2\mathbf{i} + 3\mathbf{j} + 4\mathbf{k}$.

21. Show that if $\mathbf{a} = \mathbf{b} + \lambda\mathbf{c}$, for some scalar λ, then $\mathbf{a} \times \mathbf{c} = \mathbf{b} \times \mathbf{c}$.

22. Find the area of the parallelogram with sides $\mathbf{a} = \mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$ and $\mathbf{b} = 4\mathbf{i} + 5\mathbf{j} + 6\mathbf{k}$.

(8 × 2 = 16 Marks)

SECTION – III

Answer **any six** questions from among the questions 23 to 31. These questions carry **4** marks each.

23. Find positions and number of stationary points of $f(x) = 2x^3 - 3x^2 - 36x + 2$.

24. Show that the radius of curvature at the point (x, y) on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ has magnitude $\frac{(a^4y^2 + b^4x^2)^{\frac{3}{2}}}{a^4b^4}$ and the opposite sing to y.

25. Find the area of the ellipse with semi-axes a and b using its polar coordinates.

26. Find the length of the curve $y = x^{\frac{2}{3}}$ from $x = 0$ to $x = 2$.

27. Sum the series $S(x) = \frac{x^4}{3(0!)} + \frac{x^5}{4(1!)} + \frac{x^6}{5(2!)} + \ldots$

28. Determine the range of values of z for which the complex power series $P(z) = 1 + \frac{z}{2} + \frac{z^2}{4} + \frac{z^3}{8} + \ldots$ is convergent.

29. Find the angle between the vectors $\mathbf{a} = \mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$ and $\mathbf{b} = 2\mathbf{i} + 3\mathbf{j} + 4\mathbf{k}$.

30. Find the volume of the parallelopiped with sides $\mathbf{a} = \mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$, $\mathbf{b} = 4\mathbf{i} + 5\mathbf{j} + 6\mathbf{k}$ and $\mathbf{c} = 7\mathbf{i} + 8\mathbf{j} + 10\mathbf{k}$.

31. Find the minimum distance from the point P with coordinates (1, 2, 1) to the line $\mathbf{r} = \mathbf{a} + \lambda\mathbf{b}$, where $\mathbf{a} = \mathbf{i} + \mathbf{j} + \mathbf{k}$ and $\mathbf{b} = 2\mathbf{i} - \mathbf{j} + 3\mathbf{k}$.

(6 × 4 = 24 Marks)

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SECTION – IV

Answer **any two** questions from among the questions 32 to 35. These questions carry **15** marks each.

32. (a) State and prove Mean Value Theorem.

(b) What semi-quantitative results can be deduced by applying Rolle's Theorem to the following functions

(i) $\sin x$

(ii) $\cos x$

(iii) $x^2 - 3x + 2$

(iv) $x^2 + 7x + 3$.

33. (a) Find the surface area of a cone formed by rotating about the x-axis the line $y = 2x$ between $x = 0$ and $x = h$.

(b) Evaluate $\int_0^\infty \frac{x}{(x^2 + a^2)^2} dx$.

34. Expand $f(x) = \cos x$ as a Taylor series about $x = \frac{\pi}{3}$.

35. The vertices of a triangle ABC have position vectors **a**, **b** and **c** relative to some origin O. Find the position vector of the centroid G of the triangle.

(2 × 15 = 30 Marks)

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