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Reg. No. : ....

Name : ....

First Semester B.Sc. Degree Examination, November 2018
First Degree Programme under CBCSS
COMPLEMENTARY COURSE FOR PHYSICS
MM 1131.1 : Mathematics I – Differentiation and Analytic Geometry
(2014-2017 Admissions)

Time : 3 Hours Max. Marks : 80

SECTION – I

All the **first ten** questions are **compulsory**. They carry **1 mark each** :

1. Write down the parametric equation of a cycloid.

2. If y = f(x), then the instantaneous rate of change of y with respect to x, when $x = x_0$ is __________

3. State mean value theorem.

4. Write an example for a homogenous function of degree '3' in two variables.

5. State reflection property of an ellipse.

6. $\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^{2x} =$ __________

7. Write down the local linear approximation of f(x) at x_0.

8. $\tanh^{-1}\left(\frac{1}{2}\right) =$ __________

9. Write down the natural domain for the function $f(x, y) = \frac{\sqrt{4-x^2}}{y^2+3}$.

10. Write the domain and range of *l*nx.

P.T.O.

SECTION – II

Answer **any 8** questions from among the questions **11** to **22**. These questions carry **2** marks **each** :

11. Draw the velocity versus time curve for a particle with velocity V_0 at time $t = 0$ and moving with constant acceleration.

12. Find $\lim_{t \to \infty} \frac{3t^2 + 2t}{5t^3 + 6}$.

13. What can you say about the continuity of the function $f(x) = \frac{x^2 + 25}{(x^2 - 7x + 12)}$?

14. Is the graph of $f(x) = |x|$ differentiable at $x = 0$. Prove your claim.

15. Find K if the curve $y = x^2 + k$ is a tangent to the line $y = 2x$.

16. Show that $y = x \sin x$ is a solution of $y'' + y = 2\cos x$.

17. Use implicit differentiation and find $\frac{d^2y}{dx^2}$ if $x^3 y^3 - 4 = 0$.

18. If $x^2 + y^2 = 1$, where x and y are functions of 't' and $\frac{dx}{dt} = 1$, find $\frac{dy}{dt}$ when $(x, y) = \left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$.

19. Find the local linear approximation of $\frac{1}{x}$ at $x_0 = 2$.

20. Find an interval [a, b] on which $f(x) = x^4 + x^3 - x^2 + x - 2$ satisfies the hypothesis of Rolle's theorem.

21. Find the equation of the parabola with vertex at (1, 1) and directrix $y = -2$.

22. Find the Jacobian $\frac{\partial(x, y)}{\partial(u, v)}$ if $x = 4u + v$, and $y = 5u - 3v$.

SECTION – III

Answer **any 6** questions from among the questions **23** to **31**. These questions carry **4** marks **each** :

23. Suppose that a ball is thrown vertically upward so that the height (in feet) of the ball above the ground 't' seconds after its release is modelled by the function $S(t) = -16t^2 + 29t + 6,\ 0 \leq t \leq 2$.

    a) Determine the instantaneous velocity of the ball at time t = 0.5 seconds.

    b) What is the velocity of the ball just before impacting the ground at time t = 2s ?

24. Show that $\tanh^{-1}x = \frac{1}{2}\log\left(\frac{1+x}{1-x}\right)$ and evaluate $\int_0^{1/2}\frac{dx}{1-x^2}$.

25. Show that the Maclaurin series for cos x converges to cosx for all x.

26. Find the radius of convergence and interval of convergence of the series $\sum_{k=0}^{\infty}\frac{(-1)^k x^k}{k!}$.

27. Verify Eulers theorem for the Homogenous function $u = x^3 - 2x^2y + 3xy^2 + y^3$.

28. Find the level curves of the function $f(x, y) = xy$.

29. Find the equation of the hyperbola with vertices $(0, \pm 3)$ and assymptotes $y = \pm x$.

30. A galss of lemonade with a temperature of 40°F is left to sit in a room whose temperature is a constant 70°F. If the temperature T of the lemonade reaches 52°F in 1 hour, then T is modeled by the equation $T = 70 - 30\,e^{-0.5t}$, where T is in °F and t is in hours.

    a) Find the rate of change of temperature with respect to time.

    b) Find the average temperature T_{ave} of the lemonade over the first 5 hours.

31. Find the absolute maximum and minimum values of $f = x^3 - 3x - 2$ in $(0, \infty)$ and state where these occurs.

SECTION – IV

Answer **any two**. These question carry **15** marks **each** :

32. a) A closed cylindrical can is to hold 1 liter of liquid. How should we choose the height and radius of the can to minimize the amount of material needed to manufacture the can ?

b) Use Lagrange's multipliers to find the maximum to find the maximum and minimum values of the function $f(x, y) = 3x + 4y$ on the circle $x^2 + y^2 = 1$.

c) Assume that oil spilled from a ruptured tanker spreads in a circular pattern whose radius increases at a constant rate of 2 ft/s. How fast is the area of the spill increasing when the radius of the spill is 60 ft.

33. a) If $u = \frac{xy}{x+y}$, ST $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = u$

b) If $f(x, y) = y^3e^{-5x}$ find $f_{xy}(0, 1)$.

c) If $z = \sqrt{xy + y}$, $x = \cos\theta$, $y = \sin\theta$, Use chain rule to find $\frac{dz}{d\theta}$ at $\theta = \pi/2$.

34. a) Find the slope of the sphere $x^2 + y^2 + z^2 = 1$ in z – direction at the point $\left(2/3, 1/3, -2/3\right)$.

b) Prove that the equation to the tangent to the hyperbola $x^2/a^2 - y^2/b^2 = 1$ at the point (x_0, y_0) on it is $\frac{xx_0}{a^2} - \frac{yy_0}{b^2} = 1$.

c) Describe the graph of the equation $y^2 - 8x - 6y - 23 = 0$.

35. a) Sketch the graph of $\gamma = \frac{6}{2 + \cos\theta}$ in polar coordinates.

b) State Kepler's first, second and third laws.

c) Sketch the graph of the ellipse $x^2/16 + y^2/9 = 1$ showing its focii.
