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Reg. No. : ....

Name : ....

Sixth Semester B.Sc. Degree Examination, April 2019
First Degree Programme under CBCSS
MATHEMATICS
Core Course
MM 1642 : Linear Algebra
(2013 Admission)

Time : 3 Hours Max. Marks : 80

SECTION – I

All the **first 10** questions are **compulsory**. They carry **1** mark **each**.

1. When do we say that two matrices are equal ?
2. Find $\begin{bmatrix} 1 & 2 \\ 3 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 & 1 \\ 2 & 1 \end{bmatrix}$
3. What is a consistant system of equations ?
4. Define orthogonal matrices.
5. Define kernel of a linear transformation.
6. What is a linear isomorphism ?
7. Define ordered basis.
8. If f = (1 6 4 3 5 2) and g = (2 6 5 3 1 4), find $g_\circ f$.
9. What is a transposition ?
10. Write an example of a tridiagonal matrix.

SECTION – II

Answer **any 8** questions from among the questions **11** to **22**. These questions carry **2** marks **each**.

11. Write out the 3×3 matrix whose entries are given by $x_{ij} = i + j$.
12. Prove that the addition of matrices is commutative.

P.T.O.

13. If A is orthogonal, prove that A' is also orthogonal.

14. Prove that every non-zero matrix A can be transformed to a Hermite matrix by means of elementary row operations.

15. Define left inverse and right inverse of a matrix A.

16. If A and B are invertible matrices, prove that $(AB)^{-1} = B^{-1}A^{-1}$.

17. Prove that a 2×2 matrix is orthogonal if and only if it is of the form $\begin{bmatrix} a & b \\ -b & a \end{bmatrix}$ or $\begin{bmatrix} a & b \\ b & -a \end{bmatrix}$.

18. Determine which of the following mappings are linear ?

    a) $f(x, y, z) = (y, z, 0)$ b) $g(x, y, z) = (|x|, z, 0)$

19. Let $f : R^3 \to R^2$, be given by $f(x, y, z) = (2x - 3y + z, 3x - 2y)$. Find the matrix off relative to the natural ordered basis of R^3 and R^2.

20. Prove that transition matrices are invertible.

21. Define determinantal mapping.

22. Prove that a scalar λ is an eigenvalue of an n×n matrix A if and only if $\det(A - \lambda I_n) = 0$.

SECTION – III

Answer **any 6** questions from among the questions **23** to **31**. These questions carry **4** marks **each**.

23. Write a short note on an application of matrices.

24. Explain the effect of left multiplication by the matrix $\begin{bmatrix} 0 & \alpha & 1 \\ 0 & \beta & 0 \\ 1 & \gamma & 0 \end{bmatrix}$.

25. Reduce to row-echelon form of the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \\ 5 & 5 & 8 \end{bmatrix}$.

26. Find the inverse of $\begin{bmatrix} 1 & 2 & 3 \\ 1 & 3 & 4 \\ 1 & 4 & 4 \end{bmatrix}$.

27. If $f : R^4 \to R^3$ is defined as f(a, b, c, d) = (a + b, b – c, a + d), find a basis for Im f.

28. Prove that a linear mapping is completely and uniquely determined by its action on basis.

29. Consider the linear mapping $f : R^2 \to R^3$ given by f(x, y) = (x + 2y, 2x – y, –x). Determine the matrix of f

    a) relative to the natural ordered basis

    b) relative to the basis {(0, 1), (1, 1)} and {(0, 0, 1), (0, 1, 1), (1, 1, 1)}.

30. Find the determinant of $A = \begin{bmatrix} 1 & -1 & 2 & 3 \\ 2 & 2 & 0 & 2 \\ 1 & 2 & 3 & 0 \\ 5 & 3 & 2 & -1 \end{bmatrix}$.

31. Prove that every connected graph has at least one spanning tree.

SECTION – IV

Answer **any 2** questions from among the questions **32** to **35**. These questions carry **15** marks **each**.

32. Given $A = \begin{bmatrix} 1 & 2 & -1 & -2 \\ -1 & -1 & 1 & 1 \\ 0 & 1 & 2 & 1 \end{bmatrix}$, find P and Q such that PAQ = N, where N is the normal form of A.

33. State and prove dimension theorem.

34. Prove that the rank of a linear mapping f is same as the rank of any matrix that represents f.

35. Find the limiting value of the sequence $2, 2 + \frac{1}{2}, 2 + \frac{1}{2 + \frac{1}{2}}, 2 + \frac{1}{2 + \frac{1}{2 + \frac{1}{2}}}, \ldots$.