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Reg. No. : ....

Name : ....

Sixth Semester B.Sc. Degree Examination, April 2019
First Degree Programme under CBCSS
MATHEMATICS
Core Course
MM 1641 : Real Analysis – II
(2014 Admission Onwards)

Time : 3 Hours Max. Marks : 80

SECTION – I

All the first **10** questions are **compulsory**. **Each** carries **1** mark.

1. The function $g:\mathbb{R}\to\mathbb{R}$ defined by g(x) = 2x for x rational, and g(x) = x + 3 for x irrational is continuous at x = ______________

2. Give an example for a function on [0, 1] that is discontinuous at every point of [0, 1] but | f | is continuous on [0, 1].

3. Find the points at which the function f(x) = | x | + | x + 1 | is not differentiable.

4. Using L'Hospital's Rule, find $\lim\limits_{x\to 1}\dfrac{\ln x}{x-1}$.

5. Define a convex function on an interval $I\subseteq\mathbb{R}$.

6. Let $g(x)=\left|x^3\right|, x\in\mathbb{R}$. Find $g'(x)$ for $x\neq 0$.

7. The norm of the partition P = (0, 1.5, 2, 3.4, 4) is ____________

8. If $F(x)=\frac{1}{2}x^2$ for all $x\in[a,b]$, is the antiderivative of f on [a, b], evaluate $\int_a^b f$.

9. Define a step function.

10. If J = [c, d] is a subinterval of [a, b] and $\varphi_J(x) = 1$ for $x\in J$ and $\varphi_J(x) = 0$, elsewhere in [a, b], then evaluate $\int_a^b \varphi_J$.

P.T.O.

SECTION – II

Answer **any 8** questions from this Section. **Each** question carries **2** marks.

11. Show that the Dirichlet's function defined on $\mathbb{R}$ by, $f(x)=\begin{cases}1, & \text{if } x \text{ is rational}\\ 0, & \text{if } x \text{ is irrational}\end{cases}$ is not continuous at any point on $\mathbb{R}$.

12. If $I=[a, b]$ is a closed and bounded interval and $f : I \to \mathbb{R}$ is continuous on I, prove that f is bounded on I.

13. Show that the function $f(x) = \sin x$ is continuous on $\mathbb{R}$.

14. If $m \in \mathbb{Z}, n \in \mathbb{N}$ and $x > 0$, prove that $x^{\frac{m}{n}} = \left(x^m\right)^{\frac{1}{n}}$.

15. If f and g are differentiable at c, show that f g is differentiable at c and $(fg)'(c) = f'(c)g(c) + f(c)g'(c)$.

16. Find $f'(0)$, if $f(x)=\begin{cases}x^2 \sin\left(\frac{1}{x}\right), & \text{for } x \neq 0\\ 0 & \text{for } x = 0\end{cases}$

17. If $f : I \to \mathbb{R}$ has a derivative at $c \in I$, show that f is continuous at c.

18. Explain briefly the tagged partition of a closed interval [a, b].

19. If f, $g \in R[a,b]$ show that $f + g \in R[a,b]$.

20. State and prove boundedness theorem for Riemann integral.

21. Show that the set Q_1 of rational numbers in [0, 1] is a null set.

22. If c is an interior point of an interval I at which $f : I \to \mathbb{R}$ has an extremum and if $f'(c)$ exists, then prove that $f'(c) = 0$.

SECTION – III

Answer **any 6** questions from this Section. **Each** question carries **4** marks.

23. State and prove Maximum Minimum Theorem.

24. State and prove Caratheodry Theorem.

25. If $f(x) = \begin{cases} 2, & 0 \le x < 1 \\ 1, & \text{if } 1 \le x \le 2 \end{cases}$. Show that $f \in R[0, 2]$ and evaluate the integral.

26. State and prove squeeze theorem.

27. Let $I \subseteq \mathbb{R}$ be an interval and let $f : I \to \mathbb{R}$ be monotone on I. Then prove that the set of points $D \subseteq I$ at which f is discontinuous is a countable set.

28. State and prove Rolle's Theorem.

29. Let $g : \mathbb{R} \to \mathbb{R}$ be defined by $g(x) = x + 2x^2 \sin\left(\frac{1}{x}\right)$, for $x \neq 0$ and $g(0) = 0$.

    Show that g is not monotonic in any neighborhood of 0.

30. Prove that $1 - \frac{1}{2}x^2 \le \cos x$ for all $x \in \mathbb{R}$.

31. If F, G are differentiable on [a, b], and $f = F', g = G'$ belongs to R[a, b], then prove that $\int_a^b fG = FG/_a^b - \int_a^b Fg$.

SECTION – IV

Answer **any 2** questions from this Section. **Each** question carries **15** marks.

32. a) State and prove location of roots theorem.

    b) State and prove continuous inverse theorem.

33. a) State and prove Cauchy criterion for Riemann integrability.

    b) If $f : [a, b] \to \mathbb{R}$ is continuous on [a, b], then prove that $f \in R[a, b]$.

34. a) State and prove additivity theorem.

    b) If f and g are Riemann Integrable prove that fg is Riemann Integrable.

35. a) Let I be an open interval and let $f : I \to \mathbb{R}$ have a second derivative on I. Then prove that f is a convex function if and only if $f''(x) \ge 0$ for all $x \in I$.

    b) Use Newton's Method to find an approximate value of $\sqrt{2}$.