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Reg. No. : ....

Name : ....

Fifth Semester B.Sc. Degree Examination, December 2018
First Degree Programme Under CBCSS
Mathematics
Core Course
MM 1543 : DIFFERENTIAL EQUATIONS
(2014 Admn. Onwards)

Time : 3 Hours

Max. Marks : 80

All the first **10** questions are **compulsory**. They carry **1** mark **each**.

1. The order of the differential equation $(x^2 + 3y) \frac{dy}{dx} + (5x^3 - y^2) = 0$ is

2. State True or False : $y = e^{2x}$ is a solution of $\frac{dy}{dx} = 3y$.

3. What do you mean by integral curves of a differential equation ?

4. Obtain the differential equation associated with the primitive $y = Ax^2 + Bx + C$.

5. Solve $y\frac{dy}{dx} = x$.

6. Solve the exact equation $ydx + xdy = 0$.

7. Verify that the function $y = \cos x$ is a solution of the homogeneous linear differential equation $y'' + y = 0$.

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8. Verify that $2 + \cos x + \sin x$ is not a solution of $y'' + y = 1$.

9. The auxiliary equation of $x^2 y'' + xy' - y = 0$ is

10. Verify that $2x^2 - 1$ is a solution of $y'' + 4y = 8x^2$.

Answer **any 8** questions from among the questions **11** to **22**. These questions carry **2** marks **each**.

11. Find the general solution of $y' + 2xy = 0$.

12. Solve the initial value problem $\dfrac{dy}{dx} - y \tan 2x = 0$; $y(0) = 2$.

13. Solve $\sinh x \cos y\, dx = \cosh x \sin y\, dy$.

14. Find the equation of the curve such that at each point (x, y) on the curve the slope equals the square of the distance between the point and the y-axis and the point $(-1, 2)$ is on the curve.

15. Solve $\dfrac{dy}{dx} = \dfrac{1 - 2y - 4x}{1 + y + 2x}$.

16. Make the following equation exact and hence solve : $y\, dx + (x^2 y - x)dy = 0$.

17. Find a differential equation of the form $y'' + ay' + by = 0$ for which the functions e^{-x}, e^{-2x} form a basis.

18. Verify whether the functions $x + 1$, $x - 1$ $(0 < x < 1)$ are linearly dependent or not.

19. Solve $y'' + y = 0$, $y(0) = 3$, $y(\pi) = -3$.

20. Solve $y'' + 0.25y = 0$.

21. Solve the initial value problem $y'' - 16y = 0$; $y(0) = 1$, $y'(0) = 20$.

22. Solve $x^2 y'' + xy' + y = 0$.

Answer **any 6** questions from the questions **23** to **31**. These questions carry 4 marks **each**.

23. Solve the initial value problem : $xy' - 3y = x^4(e^x + \cos x) - 2x^2$, $y(\pi) = \pi^3 e^\pi + 2\pi^2$.

24. Use Euler's method with $h = 0.1$ to solve the initial value problem

$$\frac{dy}{dx} = x^2 + y^2 \text{ with } y(0) = 0 \text{ in the range } 0 \le x \le 0.5.$$

25. Suppose the human population in the earth is increasing at an overall rate of approximately 2 percent per year of the population at that time. Find the doubling time.

26. If φ_1 and φ_2 are solutions of $y'' + f(x)y' + g(x)y = 0$ then show that $c_1\varphi_1(x) + c_2\varphi_2(x)$ is also a solution (c_1 and c_2 are arbitrary constants).

27. By reducing to first order solve the second order differential equation $xy'' = 2y'$.

28. Solve the initial value problem $y'' + 3y = 0$; $y(0) = 2$, $y'(0) = 3\sqrt{3}$.

29. If an iron ball of weight $W = 98$ Newton stretches a spring 1.09 m, how many cycles per minute will this mass-spring system execute ? What will its motion be if we pull down the weight an additional 16 cm and let it start with zero initial velocity ?

30. Show that $y_1(x) = x^{1/2}$ and $y_2(x) = x^{-1}$ form fundamental set of solutions of $2x^2y'' + 3xy' - y = 0$. $x > 0$.

31. Solve $y'' - 3y' + 2y = 4x + e^{3x}$.

Answer **any 2** questions from among the questions **32** to **35**. These questions carry **15** marks **each**.

32. a) A tank contains 50 gallons of water with a high salt content in which 75 pounds of salt are dissolved. Beginning at time $t = 0$, water containing 3 pounds of salt per gallon flows in at the rate of 2 gallon per minute and the mixture (which is kept uniform by stirring) flows out at the same rate. When will there be 125 pounds of dissolved salt in the tank ? How much dissolved salt is in the tank at time after a long time ?

b) Solve $2xy\dfrac{dy}{dx} - y^2 + x^2 = 0$

33. Solve the following differential equations.

   a) $\sin y \, dx + \cos y \, dy = 0$

   b) $\left(1+x^2\right)\dfrac{dy}{dx} = 1+y^2$

   c) $xy' - 2y = x^3 e^x$

34. a) Find a general solution of $(4D^2 + 4D + 1)y = 0$, where D is the differential operator $\dfrac{d}{dx}$.

   b) Solve the nonhomogeneous equation $y'' - 4y' + 3y = \sin 3x \cos 2x$.

35. a) Solve by the method of variation of parameters : $\left(D^2 + 4D + 4\right)y = \dfrac{e^{-2x}}{x^2}$

   b) Solve $x^2 y'' - 4xy' + 6y = 21x^{-4}$.

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