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Reg. No. : ....

Name : ....

Fifth Semester B.Sc. Degree Examination, December 2021

First Degree Programme Under CBCSS

Mathematics

Core Course VI

MM 1542 COMPLEX ANALYSIS I

(2018 & 19 Admission)

Time : 3 Hours Max. Marks : 80

SECTION – I

(Answer the **ten** questions are compulsory. They carry **1** mark each)

1. Find the quotient $\frac{5}{(1-i)(2-i)(3-i)}$.

2. Find the conjugate of $\frac{1+2i}{1-(1-i)^2}$.

3. Find the argument of $(\sqrt{3}-i)^2$.

4. Let $S=\left\{1,\frac{1}{2},\frac{1}{3},....\right\}$ what is the boundary of S.

5. Define an analytic function.

P.T.O.

6. Write the polynomial $z^4 - 16$ is factored form.

7. Find $\log(-1)$.

8. Find the principal value of i^{2i}.

9. Compute $\int_0^1 \left(2t + it^2\right)dt$.

10. State Cauchy's integral theorem.

SECTION – II

(Answer any **eight** questions. These questions carry **2** marks each.)

11. Find z if $z^2 - 2z - 2 = 0$.

12. Evaluate $(1-i)^4$.

13. Find the absolute value of $\frac{(1+3i)(1-2i)}{3+4i}$.

14. Show that for all z, $e^{2+\pi i} = e^{-z}$.

15. Write $f(z) = \frac{z+i}{z^2+1}$ in the form $w = u(x,y) + i\, v(x,y)$.

16. Show that $f(z) = Imz$ is nowhere differentiable.

17. Find $\lim_{z \to 5} \frac{3z}{z^2 - (5-i)z - 5i}$.

18. Discuss the analyticity of the function $\frac{z}{\bar{z}+2}$.

19. Show that if v is a harmonic conjugate of u in a domain D, then uv is harmonic in D.

20. Write the polynomial $(z-1)(z-2)^3$ in the Taylor form, centred at $z=2$.

21. Show that $\tan z$ is periodic with period π.

22. Find the maximum value of $|z^2+3z-1|$ in the disk $|z|\le 1$.

23. Find all values of $(1+i)^i$.

24. Show that the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is a smooth curve by producing an admissible parametrization.

25. Evaluate $\int_\Gamma e^z dz$ along the upper half of the circle $|z|=1$ from $z=1$ to $z=-1$.

26. Compute the integral $\int_\Gamma \frac{e^z+\sin z}{z}dz$, where Γ is the circle $|z-2|=3$ traversed once in the counter clockwise direction.

SECTION – III

(Answer any **six** questions. These questions carry **4** marks each)

27. Find the complex numbers z_1 and z_2 that satisfy the system of equations

$$(1-i)z_1+3z_2=2-3i$$
$$iz_1+(1+2i)z_2=1$$

28. Write the quotient $\frac{1+i}{\sqrt{3}-i}$ in polar form.

29. Prove that $1 + w_m + w_m^2 + ... + w_m^{m-1} = 0$.

30. Suppose that $f(z)$ and $\overline{f(z)}$ are analytic in a domain D. Show that $f(z)$ is constant in D.

31. Find the partial fraction decomposition of the rational function $\frac{2z+i}{z^3+z}$.

32. Establish $\sin z_1 \cos z_2 + \sin z_2 \cos z_1 = \sin(z_1 + z_2)$.

33. Determine a branch of $f(z) = \log(z^3 - 2)$ that is analytic at $z = 0$ and find $f(0)$ and $f'(0)$.

34. Derive the identity $\tan^{-1} z = \frac{i}{2} \log\left(\frac{i+z}{i-z}\right)$.

35. Prove that if C is the circle $|z| = 3$, traversed once, then $\left| \int_C \frac{dz}{z^2 - i} \leq \frac{3\pi}{4} \right|$.

36. Determine the possible values for $\int_\Gamma \frac{1}{z-a} dz$, where Γ is any circle not passing through $z = a$, traversed once in the counter clockwise direction.

37. Compute $\int_C \frac{\sin z}{z^2(z-4)} dz$ where C is the circle $|z| = 2$ traversed once in the positive sense.

38. State and explain maximum modulus principle.

SECTION – IV

(Answer any **two** questions. These questions carry **15** marks each)

39. (a) Describe the set of points z in the complex plane that satisfies each of the following.

    (i) $|2z - i| = 4$

    (ii) $|z| = \operatorname{Re} z + 2$

    (iii) $|z - i| < 2$.

    (b) Compute the integral $\int_0^{2\pi} \cos^4 \theta \, d\theta$ by using exponential form of $\cos\theta$ and binomial formula.

40. (a) Prove that if $f(z)$ is analytic in a domain D and if $f'(z) = 0$ everywhere in D, then $f(z)$ is constant in D.

    (b) Prove that if $f(z) = u(x, y) + i\, v(x, y)$ is analytic in a domain D, then each of the functions $u(x,y)$ and $v(x,y)$ is harmonic in D. Construct an analytic function whose real part is $u(x, y) = x^3 - 3xy^2 + y$.

41. (a) Prove that $\sin z = 0$ if and only if $z = k\pi$, where k is an integer.

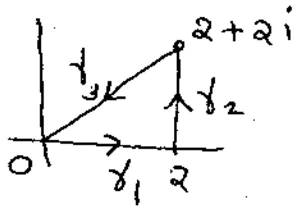
    (b) Prove that the function e^z is one to one on any open disk of radius π.

    (c) Find all numbers z such that $e^{iz} = 3$.

42. (a) Prove that the function $Log\ z$ in analytic in the domain D^* consisting of all point of the complex plane except those lying on the non positive real axis. Also $\frac{d}{dz} \log z = \frac{1}{z}$, for z in D^*.

    (b) Find all the solutions of the equation $\cos z = 2i$.

43. (a) Compute $\int_{\Gamma} \bar{z}^2 dz$ along the simple closed contour Γ given below.



(b) Suppose that the function $f(z)$ is continuous in a domain D and has an antiderivative $F(z)$ throughout D. Prove that for any contour Γ lying in D, with initial point z_I and terminal point z_T, $\int_{\Gamma} f(z)dz = F(z_T) - F(z_I)$.

44. (a) State and prove Morera's theorem.

(b) State and prove fundamental theorem of Algebra.