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Reg. No. : ....

Name : ....

Fifth Semester B.Sc. Degree Examination, December 2018
First Degree Programme under CBCSS
MATHEMATICS
Core Course V
MM 1542 : Complex Analysis – I
(2014 Admn. Onwards)

Time : 3 Hours

Max. Marks : 80

SECTION – I

All the **first ten** questions are **compulsory**. They carry **1** mark **each**.

1. Express $\frac{(5+i)(2-i)}{(1-i)}$ in the form a + ib.
2. Find the square roots of – 1.
3. Show that $\text{Im}\, z = \frac{z - \overline{z}}{2i}$.
4. Represent geometrically $\{z / z = \overline{z}\}$.
5. Find $|e^{2i}|$.
6. Define an entire function.
7. Express –1 + i in polar form.
8. Define a region in a complex plane.
9. Define radius of convergence of a power series.
10. Write the power series expansion of e^{4z}.

SECTION – II

Answer **any 8** questions from among the questions **11** to **22**. They carry **2** marks **each**.

11. Find the sum of the complex numbers 3 – i and 1 + i geometrically.
12. Find the cube roots of 8i.

P.T.O.

13. State and prove the necessary and sufficient condition for $\{z_n\}$ to converge.

14. Use Cauchy-Riemann equations to verify whether $x^2 + y^2 - 2xyi$ is analytic.

15. Does the series $\sum_{k=1}^{\infty} \frac{1}{k+i}$ converge or diverge. Justify your answer.

16. Prove that an analytic function with constant real part is a constant.

17. Evaluate $\int_C \frac{1}{z} dz$ where $C : z(t) = r\cos t + ir\sin t$, $0 \le t \le 2\pi$, $r \ne 0$.

18. Evaluate $\int_C (x^2 + iy^2) dz$ where $C : z(t) = t^2 + it^2$, $0 \le t \le 1$.

19. Find the unique real solution of $x^3 + 6x = -20$ using cubic method.

20. Is the polynomial $x^3 + 3xy^2 - x + i(3x^2y + y^3 - y)$ analytic ? Justify your answer.

21. Can a non-constant analytic polynomial be real valued ?

22. Define a smooth curve.

SECTION – III

Answer **any 6** questions from among the questions **23** to **31**. They carry **4** marks **each**.

23. Geometrically represent the following sets.

    a) $\left\{z : \frac{-\pi}{4} < \arg z < \frac{\pi}{4}\right\}$

    b) $\{z : |z - 1| < 2\}$

24. Prove $|z_1 + z_2|^2 + |z_1 - z_2|^2 = 2(|z_1|^2 + |z_2|^2)$.

25. If $\sum_{n=0}^{\infty} a_n z^n$ and $\sum_{n=0}^{\infty} b_n z^n$ converge and agree on a set of points with an accumulation point at the origin then $a_n = b_n$ for all n.

26. Find the radius of convergence of $\sum_{n=0}^{\infty} \left[1 + (-1)^n\right]^n z^n$

27. Prove that $\int_{-C} f = -\int_C f$.

28. State and prove Closed Curve Theorem.

29. a) Evaluate $\int_C (z - 2i)dz$ where C is $z(t) = t + it^2, -1 \leq t \leq 1$.

    b) Also find the above integral along the straight line from $-1 + i$ to $1 + i$.

30. a) Show that there are no analytic function $f = u + iv$ with $u(x, y) = x^2 + y^2$.

    b) Prove that $|e^z| = e^x$.

31. Suppose f is an entire function of the form $f(x, y) = u(x) + iv(y)$. Show that f is a linear polynomial.

SECTION – IV

Answer **any 2** questions from among the questions **32** to **35**. They carry **15** marks **each**.

32. Suppose $\overline{\lim}|C_k|^{\frac{1}{k}} = L$, prove that

    a) If $L = 0$, $\sum_{k=0}^{\infty} C_k z^k$ converges for all z.

    b) If $L = \infty$, $\sum_{k=0}^{\infty} C_k z^k$ converges for $z = 0$ only.

    c) If $0 < L < \infty$, set $R = \frac{1}{L}$, then $\sum_{k=0}^{\infty} C_k z^k$ converges for $|z| < R$ and diverges for $|z| > R$.

33. a) Prove that a power series is differentiable and derive the formula for its derivatives.

    b) Prove that if $f(z) = \sum_{n=0}^{\infty} c_n z^n$ has a non-zero radius of convergence, then

    $$c_n = \frac{f^n(0)}{n!} \text{ for all n.}$$

34. a) Show that the function $f(x, y) = \frac{xy(x - iy)}{x^2 + y^2}, z \neq 0$ and $f(0) = 0$ statisfies

    C. R equations at origin but it is not differentiable at origin.

    b) Suppose C is a smooth curve of length L, f is continuous on C and that $f << M$ throughout C. Then Prove that $\int_C f(z)\, dz << ML$.

35. State and prove Integral Theorem.