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F – 2465

Reg. No. : ....

Name : ....

Fifth Semester B.Sc. Degree Examination, December 2018
First Degree Programme Under CBCSS
Mathematics
Core Course
MM 1541 : REAL ANALYSIS – I
(2014 Admn. Onwards)

Time : 3 Hours

Max. Marks : 80

SECTION – 1

All the **first 10** questions are **compulsory**. They carry **1** mark **each**.

1. Define ε-nbd of $a \in R$.

2. If $S = \left\{\frac{1}{n} - \frac{1}{m} : n, m \in N\right\}$, find inf S.

3. Give an example of a set in R such that its infimum exists but has no upper bounds.

4. State Archimedean Property.

5. Give an example of a bounded sequence which is not convergent.

6. What is the limit of the sequence $\left\{\left(1+\frac{1}{n}\right)^n : n \in N\right\}$?

7. Define a contractive sequence of real numbers.

8. Find $\lim\left(\frac{2n}{n^2+1}\right)$.

9. State necessary condition for convergence of a series.

10. Define signum function. Is the sequence $(sgn(x_n))$ converges ?

P.T.O.

SECTION – 2

Answer **any 8** questions from this Section. **Each** question carries **2** marks.

11. State order properties of R.

12. State and prove Triangle Inequality.

13. Determine the set $A = \left\{ x \in R : \frac{2x+1}{x+2} < 1 \right\}$.

14. If $S = \left\{ \frac{1}{n} : n \in N \right\}$ then show that inf S = 0.

15. Show that a convergent sequence of real numbers is bounded.

16. Show that a sequence in R can have at most one limit.

17. Show that $\lim \left(\frac{\sin n}{n} \right) = 0$.

18. Show that the sequence $\left(\frac{1}{n} \right)$ is a Cauchy sequence.

19. Show that the series $\sum_{n=1}^{\infty} \frac{1}{n^2 - n + 1}$ is convergent.

20. Show that $\lim_{x \to 0} \left(x \sin \frac{1}{x} \right) = 0$.

21. Show that $\lim_{x \to 0} \frac{1}{x}$ does not exist in R.

22. Show that $\lim_{x \to 0} \frac{1}{x^2} = \infty$.

SECTION – 3

Answer **any 6** questions from this Section. **Each** question carries **4** marks.

23. State and prove Density Theorem.

24. Show that if A and B are bounded subsets of R, then $A \cup B$ is a bounded set and $\sup(A \cup B) = \sup\{\sup A, \sup B\}$.

25. State and prove Squeeze Theorem for sequence of real number.

26. State and prove Bolzano-Weierstrass Theorem.

27. Show that the series $\sum_{n=1}^{\infty} \frac{1}{n}$ is divergent.

28. Show that the series $\sum_{n=1}^{\infty} \frac{1}{n!}$ is convergent.

29. If $f : A \to R$ and if c is a cluster point of A, then show that f can have only one limit at c.

30. Show that the p-series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ converges when $p > 1$.

31. Let $f(x) = e^{1/x}$ for $x \neq 0$. Show that the right-hand limit of f(x) does not exist but the left hand limit exist and equal to 0.

SECTION – 4

Answer **any 2** questions from this Section. **Each** question carries **15** marks.

32. Show that there exists a positive real number x such that $x^2 = 2$.

33. i) State and prove Nested Intervals Property of real numbers.
    ii) State and prove Monotone convergence theorem.

34. i) Show that a sequence of real numbers is convergent if and only if it is a Cauchy sequence.
    ii) Show that every contractive sequence is a Cauchy sequence and is convergent.

35. i) Show that the geometric series $\sum_{n=1}^{\infty} r^n$ converges when $|r| < 1$.
    ii) State and prove sequential criterion theorem.