(Pages : 4) **M – 1456**

Reg. No. : ....

Name : ....

Fifth Semester B.Sc. Degree Examination, December 2021

First Degree Programme under CBCSS

Mathematics

Core Course V

MM 1541 : REAL ANALYSIS – I

(2018 and 2019 Admission)

Time : 3 Hours Max. Marks : 80

SECTION – I

All the **first 10** questions are compulsory. They carry **1** mark each.

1. State the axiom of completeness.

2. Give an example of infinite countable set.

3. State the Supremum property of R.

4. Find the limit points of the set $A = \left\{\frac{1}{n}; n \in N\right\}$.

5. Find $\lim_{n\to\infty} \frac{n}{2n+1}$.

6. What does it mean to say that a sequence (a_n) converges?

7. Give an example of a sequence which is bounded but not convergent.

8. Give an example of an open set which is not an interval.

P.T.O.

9. Find the closure of $B = \left\{ \frac{1}{3^n} n \in N \right\}$.

10. Give an example of a divergent sequence with converging subsequence.

(10 × 1 = 10 Marks)

SECTION – II

Answer any **eight** questions. Each question carries **2** marks.

11. State the cut property of the real numbers.

12. Is the set of irrational real numbers countable? Justify.

13. Define a Cauchy sequence.

14. Show by an example that every bounded real sequence may not be convergent.

15. Show that the sequence $\{a_n\}$ defined by $a_1 = \sqrt{2}$ and $a_{n+1} = \sqrt{2a_n}$ for $n \geq 1$ converges to 2.

16. Check whether the sequence $\{n + (-1)^n\}$ is monotonic or not.

17. Show that the sequence $\{(-1)^n\}$ is not convergent.

18. Show that every convergent sequence is bounded.

19. Test for convergence the series $\sum_{n=0}^{\infty} \frac{1}{2^n}$.

20. Discuss the convergence of $\sum \frac{1}{n^p}$.

21. Show that the series $\sum_{n=0}^{\infty} \frac{n}{n+1}$ diverges.

22. State the comparison test for convergence of series.

23. Show that the series $\sum_{n=0}^{\infty} \frac{1}{\sqrt{n^2+1}}$ is divergent.

24. If the series $\sum_{k=1}^{\infty} a_k$ converges, then show that $\lim_{k \to \infty} a_k = 0$.

25. Discuss the convergence of geometric series.

26. State Baire's theorem

(8 × 2 = 16 Marks)

SECTION – III

Answer any **six** questions. Each question carries **4** marks.

27. If $a, b \in R$ and $a < b$, show that there exists $r \in Q$ such that $a < r < b$.

28. Suppose that $\{a_n\}, \{b_n\}$ and $\{c_n\}$ are sequences such that $a_n \leq b_n \leq c_n$ for all $n \geq 10$, and that $\{a_n\}$ and $\{c_n\}$ both converge to 12. Then show that $\{b_n\}$ also converges to 12.

29. Define a Cantor set.

30. State and prove the Archimedean property of R.

31. Show that the sequence $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, ...\frac{n}{n+1}, ...$ is strictly monotone and bounded.

32. Show that sub sequences of a convergent sequence converge to the same limit as the original sequence.

33. Show that the union of arbitrary collection of open sets is open.

34. Show that the sequence $\left\{\frac{2n+3}{3n-2}\right\}$ is convergent.

35. Every convergent sequence is a Cauchy sequence.

36. Prove that Cauchy sequences are bounded.

37. Show that the series $\sum_{n=1}^{\infty} \frac{cos(n)}{n^2}$ is convergent.

38. Prove that A set $E \subset R$ is connected if and only if whenever $a < c < b$ with $a, b \in E$ then $c \in E$.

(6 × 4 = 24 Marks)

SECTION – IV

Answer any **two** questions. Each question carries **15** marks.

39. (a) Show that the open interval (0, 1) $= \{x \in R : 0 < x < 1\}$ is uncountable.

    (b) *State and prove nested interval property of real numbers*

40. (a) State and prove Cauchy Condensation Test.

    (b) Prove that every bounded infinite set has at least one limit point.

41. (a) Establish Cauchy criteria for convergence of sequence of real numbers.

    (b) If (a_n) and (b_n) are two sequences of real numbers converge to a and b respectively, then show that $lim(a_n + b_n) = lim\, a_n + lim\, b_n = a + b$.

42. (a) Let $x_1 = 2$ and $x_{n+1} = \frac{1}{2}\left(x_n + \frac{2}{x_n}\right)$ for $n \geq 1$. Show that x_n convergence and find its limit.

    (b) If the series $\sum_{n=1}^{\infty} |a_n|$ converges, then show that $\sum_{n=1}^{\infty} a_n$ converges as well. Is the converse true? Justify.

43. (a) *State and prove Monotone convergence theorem.*

    (b) Show that $\sum_{n=1}^{\infty} \frac{1}{n^2}$ converges

44. State and prove Heine - Borel theorem.

(2 × 15 = 30 Marks)

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