(Pages : 4) **K – 2373**

Reg. No. : ....

Name : ....

Third Semester B.Sc. Degree Examination, March 2021

First Degree Programme Under CBCSS

Statistics

(Complementary Course for Mathematics)

ST 1331.1 — PROBABILITY, DISTRIBUTIONS AND THEORY OF ESTIMATION

(2015 – 2017 Admission)

Time : 3 Hours Max. Marks : 80

Use of Statistical Table and calculator is permitted.

SECTION – A

Answer **all** questions. **Each** question carries **1** mark.

1. Write the recurrence relationship for Binomial probabilities.

2. The moment generating function of a random variable X is $\frac{0.75}{1-0.2e^{t'}}$. Find the mean of X.

3. If X and Y are independent Poisson random variables, what is conditional distribution of X given $X+Y$?

4. Define convergence in probability.

5. Write the sampling distribution of the sample mean of a random sample drawn from Normal distribution.

P.T.O.

6. If the mean of a Chi square random variable is 5, compute its variance.

7. Define *t* statistic.

8. Give an example for a maximum likelihood estimator which is unbiased and sufficient.

9. Write the 95% confidence interval for the variance σ^2 of a Normal population.

10. What is the use of Cramer - Rao lower bound?

(10 × 1 = 10 Marks)

SECTION – B

Answer **any eight** questions. **Each** question carries **2** marks.

11. Obtain the variance of discrete Uniform distribution.

12. Show that exponential distribution is a special case of Gamma distribution.

13. Define Beta distribution.

14. Derive the mean of hypergeometric distribution.

15. Write Chcbyshev's inequality. Write its significance.

16. State Lindberg – Levy central limit theorem.

17. What is meant by sampling distribution?

18. Prove the reciprocal property of F distribution.

19. Write the properties of method of moments for estimation.

20. Discuss interval estimation.

21. Define consistency of an estimator. Write an example for consistentestimator.

22. Let $x_1, x_2, \ldots, x_n$ be a random sample from Normal $(\mu, 1)$. Show that $\frac{1}{n}\sum_{i=1}^{n} x_i^2$ is an unbiased estimator of $\mu^2 + 1$.

(8 × 2 = 16 Marks)

SECTION – C

Answer **any six** questions. **Each** question carries **4** marks.

23. Derive the mode of Poisson distribution.

24. Establish the lack of memory property of geometric distribution.

25. If X is a Normal random variable with mean 11 and variance 2.25, find the value of k when

    (a) $P(X > k) = 0.3$ and

    (b) $P(X < k) = 0.09$.

26. Derive the moment generating function of Gamma distribution and hence obtain its mean and variance.

27. Let X be a random variable with probability density function $f(x) = e^{-x}, x > 0$. Compute the upper bound of $P(|X - 1| \geq 4)$ using Chebyshev's inequality.

28. If X and Y are independent and identically distributed Chi square random variables with 1 degrees of freedom. Find the value of k such that $P(X + Y \geq k) = 0.5$.

29. Derive the relationship between Chi square, t and F distributions.

30. Explain the method of maximum likelihood estimation. Write its properties.

31. Consider a random sample drawn from Binomial (n, p). Derive an estimate of p using the method of moments. Also check whether the estimator is unbiased or not.

(6 × 4 = 24 Marks)

SECTION – D

Answer **any two** questions. **Each** question carries **15** marks.

32. (a) Fit a Binomial distribution for the following data and find the expected frequencies.

Values	0	1	2	3	4
Frequency	8	32	34	24	5

(b) Derive Poisson distribution as a limiting form of Binomial distribution.

33. (a) Derive the even order central moments of Normal distribution.

(b) Let X be a Normal random variable with mean 50 and variance 16. Compute the probabilities of

(i) $X < 60$

(ii) $X > 60$

(iii) $32 < X < 48$

(iv) $52 < X < 58$

34. (a) State and prove Bernoulli's law of large numbers.

(b) Check whether the weak law of large numbers holds good for the sequence $\{X_n\}$ of independent and identically distributed random variables with $P\left(X_n = \frac{1}{\sqrt{n}}\right) = \frac{3}{4}$ and $P\left(X_n = \frac{1}{\sqrt{n}}\right) = \frac{1}{4}$.

35. (a) Explain the method of point estimation.

(b) Derive the confidence interval for the mean of a Normal population when the population standard deviation is known and when it is unknown.

(2 × 15 = 30 Marks)
