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F – 4049

Reg. No. : ....

Name : ....

Third Semester B.Sc. Degree Examination, January 2019
First Degree Programme under CBCSS
Complementary Course for Mathematics
ST1331.1 : PROBABILITY, DISTRIBUTION AND THEORY OF ESTIMATION
(2014 Admn. Onwards)

Time : 3 Hours   Max. Marks : 80

SECTION – A

Answer **all** questions, **each** carrying **1** mark.

1. What is the relation between mean and variance of binomial distribution ?
2. State reproductive property of Ψ^2 distribution.
3. Name the discrete distribution for which mean and variance have the same value.
4. State the condition for which poisson distribution is a limiting case of the binomial distribution.
5. Define the term statistic.
6. Give an example of an estimator which is unbiased and consistent.
7. What is the relation between student's t and F distribution ?
8. What is a point estimate ?
9. State desirable properties of a good estimator.
10. What is the reciprocal property of F distribution. **(10×1=10 Marks)**

P.T.O.

SECTION – B

Answer **any 8** questions, **each** carrying **2** marks.

11. What is meant by lack of memory property ? Mention any distribution which possess this property.

12. In five tossings of a fair coin, find the chance of getting 3 heads ?

13. With the usual notations find p for a binomial variate X, if n = 6 and 9P(X = 4) = P (X = 2).

14. Show that $\frac{1}{n}\sum_{i=1}^{n} X_i^2$ is unbiased for $\mu^2 + 1$ in the case of a normal population N $(\mu,1)$.

15. What are the properties of MLE ?

16. A symmetric die is thrown 600 times. Find the lower bound for the probability of getting 80 to 120 sixes.

17. Examine sufficiency of $\sum_{i=1}^{n} X_i^2$ for σ^2 in the N$(0, \sigma^2)$ distribution.

18. Given the frequency function.

    $$f(x, \theta) = \frac{e^{-x^2/2\theta^2}}{\theta\sqrt{2\pi}}, -\infty < x < \infty$$

    find the maximum likelihood estimator for θ.

19. Find the mode of a binomial distribution with p = ½ and n = 7.

20. Let X be a random variable with E(X) = 3 and V(X) = 2. Find 'h' such that

    $P\{ |x - 3| < h\} \geq 0.99$.

21. Let X_i be independent random variables assuming values 'i' and '–i' with equal probabilities whether WLLN hold for $\{X_n\}$.

22. State Lindberg – Levy central limit theorem. **(8×2=16 Marks)**

SECTION – C

Answer **any 6** questions, **each** carrying **4** marks.

23. State and prove Chebychev's inequality.

24. Let $\{X_k\}$ be mutually independent and identically distributed random variables with mean μ and finite variance, if $S_n = X_1 + X_2 + ... + X_n$, prove that the law of large numbers does not hold for the sequence $\{S_n\}$.

25. If X and Y are independent poisson random variable. Find the conditional distribution of $(X/X+Y)$.

26. Define consistency of an estimator. And show that t_n is consistent for θ if

    $E(t_n) \to \theta$ and $V(t_n) \to 0$ as $n \to \infty$.

27. A random sample $(X_1, X_2, X_3, X_4, X_5)$ is drawn from a normal population with unknown mean μ and variance σ^2. The following estimators are prepared :

$$T_1 = \frac{\sum_{i=1}^{5} X_i}{5}, \quad T_2 = \frac{X_1 + X_2}{2} + X_3, \quad T_3 = \frac{2X_1 + X_2 + 3X_3}{3}$$

    Are T_1, T_2 and T_3 are unbiased ? Also find the most efficient estimator among them.

28. If X_1, X_2 and X_3 are independent observations from a univariate normal population with mean 0 and unit variance. Find the sampling distribution of

    1) $u = \dfrac{\sqrt{2}\, X_3}{\sqrt{X_1^2 + X_2^2}}$

    2) $v = \dfrac{3X_4^2}{X_1^2 + X_2^2 + X_3^2}$

29. Obtain the 95% confidence interval for the mean of a normal population N (μ, σ) when σ known.

30. Explicate poisson distribution and its properties.

31. What is Fisher-Neymann factorization criteria on sufficiency. **(6×4=24 Marks)**

SECTION – D

Answer **any 2** questions, **each** carries **15** marks.

32. Find the MLE for random sampling from a normal population N (μ, σ^2) for :

    i) Population mean μ when the population variance σ^2 is known.

    ii) Population variance σ^2 when the population mean μ is known.

    iii) The simultaneous estimation of both the population mean and variance.

33. a) What do you mean by confidence level ?

    b) For a sample of 10 observations $\overline{X_1} = 6$ and $n_1 s_1^2 = 0.64$. For another sample of 10 observations $\overline{X_2} = 5.7$ and $n_2 s_2^2 = 0.24$. Assuming that the two samples come from normal population with the same variance σ^2 (unknown). Prepare a 95% confidence interval for $(\mu_1 - \mu_2)$.

34. If $X_1, X_2, \ldots, X_n$ are independent observations from N (0, σ^2). What is the dsbn of $Y = \dfrac{\sum X_i^2}{\sigma^2}$? Derive the distribution and state the mean, variance and useful properties.

35. i) Explain Cramer – Rao inequality.

    ii) A random sample of size n is taken from $f(x) = \theta e^{-\theta x}$, n > 0. Find the UMVUE of $1/\theta$.

(2×15=30 Marks)