



Reg. No. :

Name :

Third Semester B.Sc. Degree Examination, January 2019
First Degree Programme under CBCSS
Complementary Course for Mathematics
ST1331.1 : PROBABILITY, DISTRIBUTION AND THEORY OF ESTIMATION
(2014 Admn. Onwards)

Time : 3 Hours

Max. Marks : 80

SECTION – A

Answer **all** questions, **each** carrying **1** mark.

1. What is the relation between mean and variance of binomial distribution ?
2. State reproductive property of Ψ^2 distribution.
3. Name the discrete distribution for which mean and variance have the same value.
4. State the condition for which poisson distribution is a limiting case of the binomial distribution.
5. Define the term statistic.
6. Give an example of an estimator which is unbiased and consistent.
7. What is the relation between student's t and F distribution ?
8. What is a point estimate ?
9. State desirable properties of a good estimator.
10. What is the reciprocal property of F distribution. **(10×1=10 Marks)**

P.T.O.



SECTION – B

Answer **any 8** questions, **each** carrying **2** marks.

11. What is meant by lack of memory property ? Mention any distribution which possess this property.
12. In five tossings of a fair coin, find the chance of getting 3 heads ?
13. With the usual notations find p for a binomial variate X , if $n = 6$ and $9P(X = 4) = P(X = 2)$.
14. Show that $\frac{1}{n} \sum_{i=1}^n X_i^2$ is unbiased for $\mu^2 + 1$ in the case of a normal population $N(\mu, 1)$.
15. What are the properties of MLE ?
16. A symmetric die is thrown 600 times. Find the lower bound for the probability of getting 80 to 120 sixes.
17. Examine sufficiency of $\sum_{i=1}^n X_i^2$ for σ^2 in the $N(0, \sigma^2)$ distribution.
18. Given the frequency function.
$$f(x, \theta) = \frac{e^{-x^2/2\theta^2}}{\theta \sqrt{2\pi}}, -\infty < x < \infty$$
find the maximum likelihood estimator for θ .
19. Find the mode of a binomial distribution with $p = \frac{1}{2}$ and $n = 7$.
20. Let X be a random variable with $E(X) = 3$ and $V(X) = 2$. Find 'h' such that $P\{|x - 3| < h\} \geq 0.99$.
21. Let X_i be independent random variables assuming values 'i' and '-i' with equal probabilities whether WLLN hold for $\{X_n\}$.
22. State Lindberg – Levy central limit theorem.

(8×2=16 Marks)



SECTION – C

Answer **any 6** questions, **each** carrying **4** marks.

- 23. State and prove Chebychev's inequality.
- 24. Let $\{X_k\}$ be mutually independent and identically distributed random variables with mean μ and finite variance, if $S_n = X_1 + X_2 + \dots + X_n$, prove that the law of large numbers does not hold for the sequence $\{S_n\}$.
- 25. If X and Y are independent poisson random variable. Find the conditional distribution of $(X/X + Y)$.
- 26. Define consistency of an estimator. And show that t_n is consistent for θ if $E(t_n) \rightarrow \theta$ and $V(t_n) \rightarrow 0$ as $n \rightarrow \infty$.
- 27. A random sample $(X_1, X_2, X_3, X_4, X_5)$ is drawn from a normal population with unknown mean μ and variance σ^2 . The following estimators are prepared :

$$T_1 = \frac{\sum_{i=1}^5 X_i}{5}, \quad T_2 = \frac{X_1 + X_2}{2} + X_3, \quad T_3 = \frac{2X_1 + X_2 + 3X_3}{3}$$

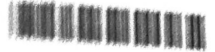
Are T_1, T_2 and T_3 are unbiased ? Also find the most efficient estimator among them.

- 28. If X_1, X_2 and X_3 are independent observations from a univariate normal population with mean 0 and unit variance. Find the sampling distribution of

1) $u = \frac{\sqrt{2} X_3}{\sqrt{X_1^2 + X_2^2}}$

2) $v = \frac{3X_4^2}{X_1^2 + X_2^2 + X_3^2}$

- 29. Obtain the 95% confidence interval for the mean of a normal population $N(\mu, \sigma)$ when σ known.
- 30. Explicate poisson distribution and its properties.
- 31. What is Fisher-Neymann factorization criteria on sufficiency. **(6x4=24 Marks)**



Answer **any 2** questions, **each** carries **15** marks.

32. Find the MLE for random sampling from a normal population $N(\mu, \sigma^2)$ for :
- Population mean μ when the population variance σ^2 is known.
 - Population variance σ^2 when the population mean μ is known.
 - The simultaneous estimation of both the population mean and variance.
33. a) What do you mean by confidence level ?
- b) For a sample of 10 observations $\bar{X}_1 = 6$ and $n_1 s_1^2 = 0.64$. For another sample of 10 observations $\bar{X}_2 = 5.7$ and $n_2 s_2^2 = 0.24$. Assuming that the two samples come from normal population with the same variance σ^2 (unknown). Prepare a 95% confidence interval for $(\mu_1 - \mu_2)$.
34. If X_1, X_2, \dots, X_n are independent observations from $N(0, \sigma^2)$. What is the dsbn of $Y = \frac{\sum X_i^2}{\sigma^2}$? Derive the distribution and state the mean, variance and useful properties.
35. i) Explain Cramer - Rao inequality.
- ii) A random sample of size n is taken from $f(x) = \theta e^{-\theta x}$, $n > 0$. Find the UMVUE of $1/\theta$. **(2×15=30 Marks)**
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