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Reg. No. : ....

Name : ....

Third Semester B.Sc. Degree Examination, March 2021

First Degree Programme under CBCSS

Statistics

Complementary Course For Mathematics

ST 1331.1 – STATISTICAL DISTRIBUTIONS

(2018 Admission)

Time : 3 Hours Max. Marks : 80

Use of statistical table and scientific calculator are permitted.

SECTION – A

Answer **all** questions. Each question carries 1 mark.

1. What are the parameters of a binomial random variable with mean 4 and variance 3?

2. Find the coefficient of variation of Poisson distribution with mean 9.

3. If X is a Poisson variable such that $P(X=1) = P(X=2)$, obtain $P(X=0)$.

4. If X and Y are independent uniform random variables over [0, 2], determine $P(X<Y)$.

5. State the distribution of $Z = X+Y$, where X and Y are independent standard normal variables.

P.T.O.

6. Define exponential distribution.

7. Define type II beta distribution.

8. What do you mean by sampling distribution?

9. State the distribution of the ratio of two independent standard normal variables.

10. Define statistic.

(10 × 1 = 10 Marks)

SECTION – B

Answer **any eight** questions. Each question carries **2** marks.

11. Define Bernoulli random variable.

12. Define hypergeometric distribution.

13. The ratio to 3 successes and 4 successes among seven independent Bernoullian trials is $\frac{1}{4}$. Find the probability of success.

14. If X follow uniform distribution with mean 1 and variance $\frac{4}{3}$, find $P(X < 0)$.

15. If $X \sim N(6, 2)$, find $P(1 < X < 3)$

16. A horizontal line of length 5 units is divided by a point chosen at random into two parts. If the length of the first part is X, find $E[X(5 - X)]$.

17. What are the advantages of Chebychev's inequality?

18. What are the conditions for Lindberg-Levy form of central limit theorem?

19. Find the mean of a random variable following chi-square distribution with n degrees of freedom.

20. A random sample of size 25 is taken from $N(1, 9)$. What is the probability that the sample mean is negative?

21. State Bernoulli's weak law of large numbers.

22. Let $X_1, X_2, ..., X_n$ are independent exponential random variables with parameter λ. Show that $X = X_1 + X_2 + ... + X_n$ follows gamma distribution.

(8 × 2 = 16 Marks)

SECTION – C

Answer **any six** questions. Each question carries **4** marks.

23. If $X \sim B(n, p)$, show that $Cov\left(\frac{X}{n}, \frac{n-X}{n}\right) = \frac{-pq}{n}$.

24. If X follows Poisson distribution with parameter unity, show that mean deviation about mean is $\frac{2}{e}$ times the standard deviation.

25. If X and Y are independent geometric variables with same parameter, find the conditional distribution of $X | X + Y$.

26. In a normal distribution 30% of the items are above 42 and 30% of the items are below 28. What are the mean and standard deviation of the distribution?

27. Derive the moment generating function of normal distribution.

28. If X follows beta distribution of the first kind with parameters p and q, show that $Y = \frac{X}{1-X}$ follow beta distribution of the second kind.

29. In a die throwing experiment using an unbiased die, X denotes the number shown by the die. Using Chebychev's inequality, prove that $P(|X - \mu| > 2.5) < 0.47$.

30. For a random sample of size 16 from $N(\mu, \sigma^2)$ population the sample variance is 16. Find a and b such that $P(a < \sigma^2 < b) = 0.6$.

31. If $X \sim N(0, 1)$, prove that $Y = X^2$ follow chi-square distribution with one degree of freedom.

(6 × 4 = 24 Marks)

SECTION – D

Answer **any two** questions. Each question carries **15** marks.

32. The numbers of printing errors per page reported in a book with 1000 pages published by a good published were noted.

No. of mistakes	0	1	2	3	4	5	6	7	8
No. of pages	626	285	65	15	6	2	1	0	0

Fit a Poisson distribution and calculate the theoretical frequencies.

33. State and prove the recurrence relation for central moments of a binomial distribution.

34. Derive the expression for central moments of a normal distribution.

35. If X is a random variable following F-distribution with (n_1, n_2) degrees of freedom. Show that $Y = \frac{1}{X}$ follows F-distribution with (n_2, n_1) degrees of freedom.

(2 × 15 = 30 Marks)