(Pages : 3) **F – 4050**

Reg. No. : .....

Name : .....

Third Semester B.Sc. Degree Examination, January 2019
First Degree Programme Under CBCSS
Mathematics
Core Course – 2
MM 1341 : ALGEBRA AND CALCULUS – 1
(2014 Admission Onwards)

Time : 3 Hours Max. Marks : 80

SECTION – I

All the **first ten** questions are **compulsory**. They carry **one** mark **each**.

1. Define characteristic of a ring.
2. Does Z_p, the ring of integers modulo p a field ? Justify.
3. Define an integral domain.
4. Show that a commutative ring with the cancellation property (under multiplication) has no zero divisors.
5. Suppose that a and b belong to an integral domain. If $a^5 = b^5$ and $a^3 = b^3$ prove that a = b.
6. What is Euler Phi-function ?
7. The set Z of integers with operation being multiplication is a group. True or False. Justify.
8. What are the distinct equivalence classes in Z/nZ ?
9. Find a unit vector $\bar{u}$ in the direction of the vector from $P_1(1, 0, 1)$ and $P_2(3, 2, 0)$.
10. Does the cross product commutative ? Justify.

(10×1=10 Marks)

P.T.O.

SECTION – II

Answer **any 8** questions from among the questions **11** to **22**. These questions carry **2** marks **each**.

11. Mention two properties of ring homomorphism.

12. Prove that cancellation laws hold in a group.

13. Show that $(b \times c) \times (c \times a) = [abc]c$.

14. Prove that Ker ϕ is a subring of a ring R.

15. Give an example of a ring homomorphism with the operations.

16. Find the Spherical co-ordinate equation for the sphere $x^2 + (y - 1)^2 + z^2 = 1$.

17. Show that the vectors $a - 2b + 3c$, $-2a + 3b - 4c$, $a - 3b + 5c$ lie in the same plane.

18. A force of 6 newtons is applied in the direction of the vector $\bar{v} = 2i + 2j - 2k$. Express the force F as a product of its magnitude and direction.

19. Find the area of a triangle with vertices P(4, 2, 0), Q(1, 3, 0) and R(1, 1, 3).

20. Find the equation of the plane through (– 3, 0, 7) and perpendicular to $\bar{n} = 5i + 2j - k$.

21. Find the angle between the planes $3x - 2y - 6z = 10$ and $4x - y + 8z = 12$.

22. Identify and sketch the quadratic surface $x^2 + y^2 = z^2$. **(8×2=16 Marks)**

SECTION – III

Answer **any 6** questions from among the questions **23** to **31**. These questions carry **4** marks **each**.

23. Prove that finite integral domain is a field.

24. Define zero divisors. State and prove one property of zero divisors.

25. If P is a prime, then prove that Z_p has no divisors of zero.

26. Find all solutions of the congruence $12x \equiv 27 \pmod{18}$.

27. State and prove the general equation of the plane.

28. Find the equation of the plane through the points $P_1(1, 2, -1)$, $P_2(2, 3, 1)$ and $P_3(3, -1, 2)$.

29. A directed line makes angles of 60° and 45° with the x-axis and y-axis respectively. Find the angle which the line makes with the z-axis.

30. If L1 : $x = 1 + 4t$, $y = 5 - 4t$, $z = -1 + 5t$ and L2 : $x = 2 + 8t$, $y = 4 - 3t$, $z = 5 + t$. Then show that the lines L1 and L2 are skew. Find the distance between the lines.

31. Find the length of the normal from the origin to the plane $2x + 6y - 3z + 21 = 0$.

(6×4=24 Marks)

SECTION – IV

Answer **any 2** questions from among the questions **32** to **35**. These questions carry **15** marks.

32. State and prove Chinese Remainder theorem.

33. i) Prove that the set G_n of nonzero elements of Z_n that are not zero divisors form a group under multiplication modulo n.

    ii) Give an example of a ring having elements a and b such that $ab = 0$ but neither a nor b is zero.

34. Define a ring. Verify that $\{a + b\sqrt{2} \mid a, b \in Z\}$ with usual addition and multiplication is a commutative ring with unity. Check whether it is a field.

35. A) Describe the surface :

    i) $z = (x - 1)^2 + (y + 2)^2 + 3$

    ii) $4x^2 + 4y^2 + z^2 + 8y - 4z = -4$

    B) Identify the surfaces :

    a) $3x^2 - 4y^2 + 12z^2 + 12 = 0$

    b) $4x^2 - 4y + z^2 = 0$.

(2×15=30 Marks)