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Reg. No. : ....

Name : ....

Third Semester B.Sc. Degree Examination, March 2021

First Degree Programme under CBCSS

Mathematics

Core Course — II

MM 1341 : ELEMENTARY NUMBER THEORY AND CALCULUS — I

(2018 Admission)

Time : 3 Hours Max. Marks : 80

SECTION I

All the questions are compulsory. Each question carries **1** mark.

1. State the Inclusion-Exclusion Principle for 3 finite sets *A*, *B* and *C*.

2. Define a Linear Diophantine Equation in two variables.

3. Define Derivative of a vector-valued function $r(t)$.

4. Evaluate the definite integral, $\int_0^2 2ti + 3t^2 j\,dt$.

5. Define Unit Normal Vector.

6. Give the formula for a_N, the normal component of acceleration for a moving particle in terms of the velocity v and the acceleration a.

P.T.O.

7. Find the natural domain of $f(x,y,z)=\sqrt{1-x^2-y^2-z^2}$.

8. Define an Open set.

9. Define Gradient of a function $f(x,y,z)$.

10. State the Extreme value theorem.

SECTION II

Answer **any eight** questions. Each question carries **2** marks.

11. State and prove the Pigeonhole Principle.

12. Express $3ABC_{sixteen}$ in base 10.

13. Find $(4076, 1024)$.

14. Prove that two integers a and b are relatively prime if and only if $[a, b]=ab$.

15. Let $r(t)=t^2 i+e^t j-(2\cos\pi t)k$. Find $\lim_{t\to 0} r(t)$.

16. Define a tangent vector and tangent line through the point P on the graph of a vector-valued function $r(t)$.

17. State the chain rule for differentiation of vector functions.

18. A particle moves through 3-space in such a way that its velocity is $v(t)=i+t\,j+t^2 k$. Find the coordinates of the particle at time t.

19. Define Level surfaces and find the level surfaces of $f(x,y,z)=z^2-x^2-y^2$.

20. Define local linear approximation to $f(x,y)$ at (x_0,y_0).

21. Consider the ellipsoid $x^2+4y^2+z^2=18$. Find an equation of the tangent plane to the ellipsoid at the point $(1,2,1)$.

22. Explain the steps to find the absolute extrema of a continuous function f of two variables on a closed and bounded set R.

SECTION III

Answer **any six** questions. Each question carries **4** marks.

23. Find the number of integers ≤ 3000 and divisible by 3, 5 or 7.

24. Prove that if a and b are positive integers, then $[a,b]=\frac{ab}{(a,b)}$.

25. Using canonical decomposition, find the LCM of 1050 and 2574.

26. Find parametric equations of the tangent line to the circular helix $x=\cos t, y=\sin t, z=t$ where $t=t_0$, and use that result to find parametric equations for the tangent line at the point where $t=\pi$.

27. If $r_1(t)$ and $r_2(t)$ are two vector functions of t, derive the expression for $\frac{d}{dt}(r_1 \times r_2)$.

28. Find $r(t)$ given that $r'(t)=(3,2t)$ and $r(1)=(2,5)$.

29. Assuming that polynomials in one variable and cosine function are continuous, show that $f(x,y)=\cos(3x^3y^4)$ is continuous everywhere. State the results used for the proof.

30. Differentiability of function $f(x,y)$ at a point implies continuity at that point. Justify the statement.

31. Suppose that $w=x^2+y^2-z^2$ and $x=\rho\sin\varphi\cos\theta, y=\rho\sin\varphi\sin\theta, z=\rho\cos\varphi$ use appropriate forms of the chain rule to find $\frac{\partial w}{\partial \rho}$ and $\frac{\partial w}{\partial \theta}$.

SECTION IV

Answer **any two** questions. Each question carries **15** marks.

32. (a) State and Prove the Fundamental Theorem of Arithmetic.

(b) Let $b \geq 2$ be an integer and b+1 integers are randomly selected. Prove that the difference of two of them is divisible by b.

33. (a) If a circle is parameterised by arc length as

$$r(s)=a\cos\left(\frac{s}{a}\right)i+a\sin\left(\frac{s}{a}\right)j\,(0\le s\le 2\pi a).$$ Find $T(s)$ and $N(s)$.

(b) Find the curvature $k(t)$ for the circular helix $x=a\cos t, y=a\sin t, z=ct$ where $a>0$.

34. Given $f(x,y)=\frac{xy}{x^2+y^2}$, find the limit of $f(x,y)$ as $(x,y)\to(0,0)$ along

(a) The x-axis

(b) The y-axis

(c) The line $y=x$

(d) The line $y=-x$

(e) The parabola $y=x^2$.

35. Explain Lagrange multipliers Method and use it to determine the dimensions of a rectangular box, open at the top, having a volume of 32 ft^3, and requiring the least amount of material for its construction. Given the surface area being $S=xy+2xz+2yz$.
