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Reg. No. : ....

Name : ....

Third Semester B.Sc. Degree Examination, October 2019

First Degree Programme under CBCSS

Mathematics

Core Course – 2

MM 1341 ELEMENTARY NUMBER THEORY AND CALCULUS I

(2018 Admission)

Time : 3 Hours Max. Marks : 80

PART – A

Answer all the **ten** are compulsory. They carry **1** mark each.

1. Multiply 1011_{two} and 101_{two}.

2. Let *f(n)* denote the number of positive integers $\leq n$ and relatively prime to it. Find *f(24)*.

3. Find the domain of *r(t)* and the value of $r(t_0)$.

   where $r(t) = \cos t\, i - 3t\, j; t_0 = \pi$

4. Find $r'(t)$ if

   $r(t) = (\tan^{-1} t)i + t\cos t\, j - \sqrt{t}k$

5. Evaluate $\int \langle te^t, \ln t\rangle dt$.

P.T.O.

6. Find the unit tangent vector to the graph of $r(t) = t^2 i + t^3 j$ at the point where $t = 2$.

7. Determine whether the statement "If $r(s)$ is parameterized by arc length, then the curvature of the graph of $r(s)$ is the length of $r'(s)$." Is true or false?

8. Find $\lim_{(x,y)\to(0,0)} \ln(1+x^2y^3)$.

9. Find $f_x(x,y), f_y(x,y)$ of $f(x,y)=\frac{1}{xy^2-x^2y}$.

10. Find the gradient of $f(x, y) = 5x^2 + y^4$ at $(4, 2)$.

(10 × 1 = 10 Marks)

PART – B

Answer **any eight** questions from this section. Each question carries **2** marks.

11. Show that $n^3 — n$ is divisible by 2.

12. Express 3014 in base eight.

13. Using recursion, evaluate (18, 30, 60, 75, 132).

14. Find the parametric equations that correspond to the given vector equation.

$$r = 3t^2 i - 2j$$

15. Evaluate the definite integral $\int_0^2 (2t\,i + 3t^2\,j)\,dt$.

16. Find the arc length of the parametric curve.

17. Find the curvature of $x = e^t \cos t$, $y = e^t \sin t$, $z = e^t$ at $t = 0$.

18. Suppose that $w = xy + yz$, $y = \sin x$, $z = e^x$. Use an appropriate form of the chain rule to find $\frac{dw}{dx}$.

19. Find the displacement of $r = t^2 i + \frac{1}{3}t^3 j$ in the interval $1 \le t \le 3$.

20. Describe the largest region on which the function $f(x, y, z) = 3x^2 e^{yz} \cos(xyz)$ is continuous.

21. Given $f(x, y) = x^3y^5 - 2x^2y + x$, find f_{xxy} and f_{yxy}.

22. Find an equation for the tangent plane to the surface $x^2 + y^2 + z^2 = 25$ at the point P(–3, 0,4).

(8 × 2 = 16 Marks)

PART – C

Answer **any six** questions from this section. Each question carries **4** marks.

23. Show that the number of leap years l after 1600 and not exceeding a given year y is given by $l = \lfloor y/4 \rfloor - \lfloor y/100 \rfloor + \lfloor y/400 \rfloor - 388$.

24. Show that "If p and $p^2 + 2$ are primes, then $p^3 + 2$ is also a prime."

25. A six-digit positive integer is cut up in the middle into two three-digit numbers. If the square of their sum yields the original number, find the number.

26. Find the escape speed in km/s for a space probe in a circular orbit that is 300 km above the surface of the Earth.

27. A particle moves along the parabola $y = x^2$ with a constant speed of 3 units per second. Find the normal scalar component of acceleration as a function of x.

28. Suppose that a particle moves along a circular helix in 3-space so that its position vector at time t is $r(t) = (4\cos t\pi)i + (4\sin \pi t)j + tk$. Find the distance traveled and the displacement of the particle during the time interval $1 \le t \le 5$.

29. Locate all relative maxima, relative minima, and saddle points, if any for the function $f(x, y) = y^2 + xy + 3y + 2x + 3$.

30. Find the point on the plane $x + 2y + 3z = 13$ closest to the point (1, 1, 1).

31. Find the dimensions of the closed right circular cylindrical can of smallest surface area whose volume is $16\pi\ cm^3$.

(6 × 4 = 24 Marks)

PART – D

Answer **any two** questions from this section. Each question carries **15** marks.

32. (a) Find the number of positive integers ≤ 3000 and divisible by 3, 5, or 7.

    (b) Every positive integer n can be written as $n = 2^a 5^b c$, where c is not divisible by 2 or 5.

    (c) Find the canonical decomposition of 2520.

33. (a) Solve the vector initial-value problem for $y'(t) = 2ti + 3t^2 j, y(0) = i - j$ by integrating and using the initial conditions to find the constants of integration.

    (b) Find the arc length of that portion of the circular helix $x = \cos t, y = \sin t, z = t$ from $t = 0$ to $t = \pi$.

    (c) Find the curvature and the radius of curvature $x = e^t \cos t, y = e^t \sin t, z = e^t$ at the point t=0.

34. A particle moves along a circular path in such a way that its x– and y–coordinates at time t are

$$x = 2 \cos t, y = 2 \sin t.$$

    (a) Find the instantaneous velocity and speed of the particle at time t.

    (b) Sketch the path of the particle, and show the position and velocity vectors at time t = $\pi/4$ with the velocity vector drawn so that its initial point is at the tip of the position vector.

    (c) Show that at each instant the acceleration vector is perpendicular to the velocity vector.

35. (a) Find $\lim_{(x,y)\to(0,0)} \tan^{-1}\left[\frac{x^2+1}{x^2+(y-1)^2}\right]$.

    (b) Let $f(x,y) = \begin{cases} -\frac{xy}{x^2+y^2} & (x,y) \neq (0,0) \\ 0 & (x,y) = (0,0) \end{cases}$ Show that $f_x(x, y)$ and $f_y(x, y)$ exist at all points (x, y).

    (c) Use the method of Lagrange multipliers to find the dimensions of a rectangle with perimeter p and maximum area. **(2 × 15 = 30 Marks)**