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Reg. No. : ....

Name : ....

Second Semester B.Sc. Degree Examination, August 2018
First Degree Programme under CBCSS
COMPLEMENTARY COURSE FOR MATHEMATICS
ST 1231.1 – Random Variables and Analysis of Bivariate Data
(2014 Admn. Onwards)

Time : 3 Hours Max. Marks : 80

SECTION – A

Answer **all** questions. **Each** carrying **one** mark.

1. Distinguish between a discrete and continuous random variable.
2. Write down the properties of a probability density function.
3. When do you say that two random variables are independent ?
4. Let X be a continuous random variable with pdf f(x) = 2x, 0 < x < 1. Find the value of P(X < 1/2).
5. Define the expectation of a continuous random variable.
6. Let X be a discrete random variable with probability mass function

x	0	1	2	3	4	5	6
P(x)	1/8	1/8	1/8	1/4	1/4	1/16	1/16

   Find E(2X + 3).

7. Define probability generating function of a random variable.
8. Explain the significance of a scatter diagram.
9. What is rank correlation ?
10. Explain the logic of curve fitting. **(10×1=10 Marks)**

P.T.O.

SECTION – B

Answer **any 8** questions. **Each** question carries **2** marks.

11. X is a discrete random variable taking values 1, 2, 3, 4 with probabilities $\frac{1}{4}$ each. Obtain its distribution function.

12. If X and Y are two discrete random variables show that $E(X+Y) = E(X) + E(Y)$.

13. What are the properties of a moment generating function ?

14. If X and Y are independent random variables show that $E(XY) = E(X)E(Y)$.

15. A continuous random variable has probability density function $f(x) = ce^{-3x}$, find c.

16. Find the characteristic function of the random variable whose distribution function is $F(x) = 1-e^{x}$.

17. An unbiased coin is tossed twice. Let X denotes the number of heads and Y denote number of heads minus number of tails. Find the point distribution of (X, Y).

18. Explain linear regression with the help of an example.

19. Explain the method of estimating the coefficients of regression using principle of least squares.

20. Define conditional distribution and conditional expectation.

21. Explain why there are two regression lines ?

22. Show that when the correlation coefficient is 1 the regression lines are parallel.

(8×2=16 Marks)

SECTION – C

Answer **any 6** questions. **Each** carries **4** marks.

23. State and prove any two properties of a characteristic function.

24. The probability mass function of a random variable X is given as follows :

x	0	1	2	3	4	5
P(x)	k^2	k/4	5k/2	k/4	$2k^2$	k^2

Find k. Write down the distribution function of X.

25. A continuous random variable X has the pdf $f(x) = Ae^{-x/5}$. Find A. Show that for any two constants a and b $P(X > a + b/X > b) = P(X > a)$.

26. The probability density function of a random vector (X, Y) is given by $f(x, y) = 2, 0 < x < y < 1$. Find the conditional mean and variance of X given Y = y.

27. State and prove Cauchy-Schwartz inequality.

28. Prove or disprove zero covariance implies independence. What can you say about the converse ?

29. Obtain the standard error of estimate of Y as given by the regression of Y on x.

30. Explain the method of fitting a curve of the form $Y = ab^x$.

31. For the regression lines $4x - 5y + 33 = 0$ and $20x - 9y = 107$, find
    a) Mean value of X and Y.
    b) The correlation between X and Y.

(6×4=24 Marks)

SECTION – D

Answer **any 2** questions. **Each** carries **15** marks.

32. If $f(x, y) = 1/72\,(2x + 3y)$; $x = 0, 1, 2;\quad y = 1, 2, 3$ is the joint probability mass function of X and Y.
    a) Find probability mass function of X = Y
    b) Find the conditional distribution of X given X + Y = 3
    c) Examine whether X and Y are independent.

33. Let X and Y have joint probability density function $f(x, y) = 3x/4$, $0 < x < y < 1$. Find the conditional distribution of X given Y = y. Also find the correlation between X and Y.

34. Differentiate between correlation and regression. Fit a curve of the form $Y = ax^b$ to the following data :

X :	3	5	7	9	11
Y :	19	48	103	174	430

Estimate Y when X = 5.

35. A) Derive the formula for the rank correlation coefficient.

B) Eight competitors in a dance contest are ranked by two judges in the following order.

Sl. No.	1	2	3	4	5	6	7	8
Judge A	3	5	4	8	7	1	2	6
Judge B	4	6	3	8	7	2	1	5

Find the rank correlation between the ranks of the judges. **(2×15=30 Marks)**