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Reg. No. : ....

Name : ....

Second Semester B.Sc. Degree Examination, December 2021

First Degree Programme Under CBCSS

Statistics

Complementary Course for Mathematics

ST 1231.1 — PROBABILITY AND RANDOM VARIABLES

(2018 and 2019 Admission)

Time : 3 Hours Max. Marks : 80

SECTION – A

Answer **all** questions, each question carries **1** mark.

1. Define sample space of a random experiment with suitable example.
2. When will you say that several events are mutually exclusive and exhaustive?
3. Mention any two drawbacks of the classical definition of probability.
4. State multiplication theorem of probability.
5. Define a priori probability.
6. Distinguish between discrete and continuous random variables.
7. Find the value of *a*, if a random variable *X* had pdf $f(x) = ae^{-\frac{x}{5}}$, $x > 0$.
8. For two random variables *X* and Y, show that $E(X + Y) = E(X) + E(Y)$
9. Show that the first central moment of a random variable is always zero.
10. Show by an example that the moment generating function of a random variable does not exist always.

(10 × 1 = 10 Marks)

P.T.O.

SECTION – B

Answer any **eight** questions, each question carries **2** marks.

11. Seven balls are distributed in two bags at random. What is the probability that the first bag contains 4 balls?

12. If two unbiased dice are thrown, what is the probability that the product of the outcomes is a prime number?

13. If A and B are mutually exclusive events and $P(A)=\frac{1}{3}$ and $P(B)=\frac{1}{4}$, find the values of $P(\overline{A}\cap B)$ and $P(\overline{A}\cup B)$.

14. Show that if two events A and B are independent,

$P(A\cap B)=P(A).P(B)$

15. Show that pairwise independence does not imply mutual independence, with the help of an example.

16. A bag contains 3 oranges and 2 apples and another bag contains 4 oranges and 3 apples. A bag is selected at random and one fruit is drawn from the selected bag. What is the probability that the selected fruit is an orange?

17. Define the distribution function of a random variable. What are its important properties?

18. The joint pdf of two random variables X and Y is

$$f(x,y)=\frac{1}{27}(x+2y),\ x=0,1,2;\ y=0,1,2$$

Find the marginal density functions of X and Y.

19. If a random variable X has pdf $f(x)=e^{x}$, $x>0$, find the distribution of $Y=3X$.

20. Show that $\mu_2=\mu_2'-(\mu_1')^2$ where μ_r represents r^{th} central moment and μ_1' represents the r^{th} raw moment.

21. Define the conditional expectation $E(X/Y)$.

22. Define the characteristic function of a random variable. What is its advantage over the moment generating function?

(8 × 2 = 16 Marks)

SECTION – C

Answer any **six** questions. **Each** question carries **4** marks.

23. State and prove addition theorem of probability.

24. A five digit number is formed with digits 0,1,2,3,4. Find the probability that the number is (a) divisible by 5 (b) an odd number.

25. The probability that a 50 year old man will be alive at 60 is 0.83 and the probability that a 45 year old woman will be alive at 55 is 0.87. What is the probability that a man, who is 50 and his wife, who is 45 will both alive 10 years hence?

26. State and prove Baye's theorem.

27. Find the value of k such that $f(x) = kx(1-x),\ 0 < x < 1$ is a pdf. Also find $P\left[X > \frac{1}{2}\right]$.

28. The joint pdf of two random variables X and Y is given by $f(x, y) = \frac{1}{8}(6 - x - y)$, $0 < x < 2,\ 2 < y < 4$

    (a) Find the marginal density functions of X and Y

    (b) Examine whether X and Y are independent.

29. If X is a random variable with density function $f(x) = \left(\frac{1}{2}\right)^x$, $x = 1,2,3\ldots$, find the pdf of $Y = X^2$.

30. Derive the relationship between the r^{th} central moment and raw moments.

31. When will you say that several random variables are mutually independent? Give an example for mutually independent events.

(6 × 4 = 24 Marks)

SECTION – D

Answer any **two** questions, each question carries **15** marks.

32. (a) Define the term 'Statistical regularity'. How the probability is defined based on this property?

    (b) A town has two doctors A and B, working independently. The probability that doctor A is available for consultation at a particular time is 0.9 and the doctor B is available with probability 0.8. What is the probability that at least one doctor is available when needed?

33. (a) Define the conditional probability of events.

    (b) For three events A, B and C. show that

    $$P(A \cup B / C) = P(A / C) + P(B / C) + P(A \cap B / C)$$

34. (a) Define probability mass function of a discrete random variable. What are its important properties?

    (b) A coin is tossed till the outcome is 'head' and the random variable X dinote the number of tosses. Write the probability mass function of X.

35. (a) Show that the moment generating function of the sum of two independent random variables is the product of their moment generating functions.

    (b) If a random variable X has pdf $f(x) = \lambda e^{-\lambda x}$, $x > 0$, find the moment generating function and hence the mean and variance of X.

(2 × 15 = 30 Marks)