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Reg. No. : ....

Name : ....

Second Semester B.Sc. Degree Examination, May 2019

First Degree Programme under CBCSS

Complementary Course for Mathematics

ST 1231.1 : PROBABILITY AND RANDOM VARIABLES

(2018 Admission)

Time : 3 Hours Max. Marks : 80

SECTION – A

Answer **all** questions. Each question carries **1** mark. :

1. Define random variable $\overline{a}$ with suitable example.
2. If two events A and B are disjoint, show that $P(A \cup B) = P(A) + P(B)$.
3. Define independence of events with example.
4. Define a posteriori probability.
5. What is the classical definition of probability?
6. Mention any two properties of probability mass function of a discrete random variable.
7. Define the probability density function of a continuous random variable.
8. Define mathematical expectation of a random variable.
9. Distinguish between raw moments and central moments.
10. For two independent random variables X and Y, show that $M_t(X + Y) = M_t(X).M_t(Y)$.

(10 × 1 = 10 Marks)

P.T.O.

SECTION – B

Answer any **eight** questions. Each question carries **2** marks. :

11. Four students are selected from 5 boys and 3 girls. What is probability that the selected group contains only boys?

12. Using axioms of probability, show that $P(\overline{A}) = 1 - P(A)$.

13. If an unbiased coin is tossed 5 times, find the probability that all of them are not head.

14. Define conditional probability of two events A and B with suitable examples.

15. If two events A and B are independent, show that the events $\overline{A}$ and $\overline{B}$ are independent.

16. When will you say that several events are mutually exclusive and exhaustive. Also give an example.

17. Distinguish between discrete and continuous random variables.

18. If a random variable X has the pdf $f(x) = kx$, $x = 1,2,3,4,5$, find the values of k. Also find $P(X \geq 4)$

19. If a random variable X has pdf $f(x) = \frac{1}{3}$, $x = 1, 2, 3$, find the distribution of the random variable $Y = 2X + 3$.

20. For a random variable X, show that $\mu_0 = 1$ and , $\mu_1 = 0$, where μ_r represents the rth central moment of X.

21. For two independent random variables, show that $E(XY) = E(X).E(Y)$.

22. Define moment generating function of a random variable. Show by an example that it does not exists always.

(8 × 2 = 16 Marks)

SECTION – C

Answer any **six** questions. Each question carries **4** marks. :

23. Define statistical regularity and explain how probability can be defined using this property.

24. A bag contains 3 red, 6 white and 7 blue balls. What is the probability that two balls drawn are white and blue?

25. A dice is biased so that the chance of happening the even number is twice as that of an odd number. If the dice is thrown two times, what is the probability that sum of the two numbers thrown is an even number?

26. A bag contains four tickets with numbers 112,121,211,222. One ticket is drawn and the number is noted. Let A_1 be the event that the first digit of the number in the drawn ticket is one. Similarly A_2 and A_3 are the events that second and third digits of the number in the drawn ticket is one. Examine whether A_1, A_2 and A_3 are independent.

27. A continuous random variable X had pdf $f(x) = Ax^2$, $0 < x < 1$. Determine the value of A and find $P\left[\frac{1}{3} < X < \frac{2}{3}\right]$.

28. The joint pdf of two random variables X and Y is

$$f(x, y) = \frac{1}{27}(x + 2y),\ x = 0, 1, 2;\ y = 0, 1, 2.$$

(a) Find the marginal density functions of X and Y

(b) Find the conditional distribution of Y given $X = x$.

29. If a random variable X has pdf $f(x) = 1$, $0 < x < 1$, find the distribution of $Y = -2\log(X)$.

30. State and prove Cauchy - Schawrtz inequality.

31. A random variable X has pdf $f(x) = \lambda e^{-\lambda x}$, $0 < x < \infty$. Find the moment generating function and hence its mean and variance.

(6 × 4 = 24 Marks)

SECTION – D

Answer any **two** questions. Each question carries **15** marks. :

32. (a) Define axiomatic definition of probability.

    (b) A man forgets the last digit of a telephone number and he dials the last digit at random.What is the probability of calling no more than three wrong numbers?

33. (a) State and prove Baye's theorem.

    (b) There are two identical boxes contains 5 white and 4 red balls, 4 white and 6 red balls. A box selected at random and a ball is drawn from it. If the drawn ball is red, what is the probability that it is from the second box?

34. (a) Define the distribution function of a random variable. What are its important properties.

    (b) A random variable X has the following probability density function

X :	–2	–1	0	1	2	3
$p(x)$:	0.1	k	0.2	$2k$	0.3	k

    Find the value of k and also find the distribution function of X.

35. (a) Define the variance of a random variable and covariance between two random variables.

    (b) If the joint density function of a bivariate random variable (X, Y) is

$$f(x, y) = \begin{cases} 2 - x - y, & 0 < x < 1, \quad 0 < y < 1 \\ 0, & \text{otherwise} \end{cases}$$

    Find the variance of X and Y and covariance between X and Y.

(2 × 15 = 30 Marks)