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Reg. No. : .....

Name : .....

Second Semester B.Sc. Degree Examination, May 2019

First Degree Programme under CBCSS

Mathematics

Foundation Course II

MM 1221 : FOUNDATIONS OF MATHEMATICS

(2014-2017 Admissions)

Time : 3 Hours Max. Marks : 80

PART – A

Answer **all** questions. Each question carries 1 mark :

1. Show that $a \equiv b \pmod{1001}$ implies $a \equiv b \pmod{7}$.
2. Is {1 , 3, 5, 7, 9, 11, 13} is a complete set $\mathbb{Z}/7\mathbb{Z}$.
3. Find the order of 2 modulo 7.
4. State Euler's theorem.
5. Find the intervals on which, $f(x) = x^3 - 3x^2 + 1$ is concave up and downwards.
6. State the Extreme value theorem.
7. Define definite integral of a function f.

8. Find the inverse of $y = \sqrt{3x^2 - 2}$..

9. Evaluate $\int_1^{-\infty} \frac{dx}{x^2}$.

10. Find the polar co-ordinates of the point whose rectangular co ordinates are $\left(-2, 2\sqrt{3}\right)$.

(10 × 1 = 10 Marks)

PART – B

Answer **any eight** questions. Each question carries 2 marks.

11. Prove that, if $\alpha \equiv b (\text{mod}\, r)$ and $a \equiv b \ (\text{mod}\, s)$, then $a \equiv b (\text{mod}\ [r,s]$.

12. Show that in $\mathbb{Z}/m\mathbb{Z}$, [a] is unit if and only if $(a, m) = 1$.

13. Define the terms in Trial division : (a) Wheel (b) Spokes.

14. State Fermat's theorem in terms of congruence classes and in terms of divisibility.

15. Find the points of inflection of x^4.

16. Find the absolute extrema of $f(x) = 6x^{\frac{3}{4}} - 3x^{\frac{1}{3}}$ on [-1, 1].

17. Evaluate $\int \frac{\cos x}{\sin^2 x} dx$.

18. Find the volume of the solid generated by revolving the region enclosed by the curves $y = \sqrt{x}, y = 2, y = 0$ about the y-axis.

19. Solve $\frac{e^x - e^{-x}}{2} = 1$ for x.

20. Evaluate $\lim_{x\to 0}+\left(\frac{1}{x}-\frac{1}{\sin x}\right)$.

21. Find the area enclosed in first quadrant by the cardioid $r=1-\cos\theta$.

22. Express $r=2+\cos\frac{5\theta}{2}$, parametrically.

(8 × 2 = 16 Marks)

PART – C

Answer **any six** questions. Each question carries 4 marks.

23. Show that $2^{340}\equiv 1(\text{mod } 341)$.

24. Show that if p is a prime and a is not divisible by p, then the order of a modulo p divides $p-1$.

25. Find the order of [2], [3], [4], [5] in $\mathbb{Z}/7\mathbb{Z}$.

26. Find the absolute the extrema of $f(x)=\frac{1}{x^2-x}$ on (0, 1)

27. Find the volume of the right pyramid whose altitude is *h* and whose base is a square with sides of length *a*.

28. Find the length of the curve $y=x^{\frac{3}{2}}$ from (1,1) to (2,2\ $\sqrt{2}$).

29. Prove that $\sinh^{-1}x=$ in $(x+\sqrt{x^2+1})$.

30. Evaluate $\int_{-\infty}^{\infty}\frac{dx}{1+x^2}$.

31. Find the area enclosed by the rose $r=\cos 2\theta$.

(6 × 4 = 24 Marks)

PART – D

Answer **any two** questions. Each question carries 15 marks.

32. (a) Write the round by round match schedule with 6 players.

(b) Define Euler phi function and show that $\phi(p^n) = p^n\left(1-\frac{1}{p}\right)$.

(c) Prove that if 'a' has order e (mod m), then the order of a^d modulo m is $e/(d, e)$ where (d, e) is the greatest common divisor of d and e.

33. (a) Briefly explain the encoding and decoding procedures in RSA codes.

(b) Evaluate $\int_0^2 x(x^2+1)^3\,dx$ and $\int_0^{\frac{\pi}{8}} \sin^5 2x \cos 2x\,dx$.

34. (a) Find the area of the region enclosed $x = y^2$ and $y = x - 2$ by integrating with respect to x. Verify the result by integrating with respect to y also.

(b) State Arc Length formula for parameterized curves and use it to fin the circumference of a circle of radius a.

35. (a) Evaluate $\int \frac{x^2+x-2}{3x^2-x^2+3x+1}dx$.

(b) Sketch the graph of $r = \frac{6}{z+\cos\theta}$, in polar co-ordinates.

(2 × 15 = 30 Marks)
