(Pages : 4) **H – 1519**

Reg. No. : ....

Name : ....

Third Semester B.Sc. Degree Examination, October 2019

First Degree Programme under CBCSS

Complementary Course for Chemistry and Polymer Chemistry

MM 1331.2 : MATHEMATICS III – VECTOR ANALYSIS AND THEORY OF EQUATIONS

(2014-2017 Admissions)

Time : 3 Hours Max. Marks : 80

PART – A

Answer **all** questions. **Each** question carries **1** mark.

1. What is your observation about imaginary roots of a polynomial equation?

2. State Des-Carte's rule of signs of a polynomial equation.

3. Form a rational cubic equation whose roots include 3 and $\sqrt{2}$.

4. Give an example for transcendental equation.

5. Define the continuity of a vector function.

6. Write down the condition that $Mdx + Ndy + Pdx$ is exact.

7. Define $div\phi$.

8. Define the line integral of F along C.

P.T.O.

9. If $\bar{r} = xi + yj + zk$, evaluate $curl\ \bar{r}$.

10. State Gauss divergence theorem.

(10 × 1 = 10 Marks)

PART – B

Answer any **eight** questions. **Each** question carries **2** marks.

11. Solve the equation $2x^3 - 9x^2 - 27x + 54 = 0$, given that its roots are in GP.

12. Solve the equation $x^4 - 2x^3 + 4x^2 + 6x - 21 = 0$, given that two of its roots are equal in magnitude and opposite in sign.

13. Solve $2x^3 - 7x^2 + 36 = 0$ given that the difference between two of the roots is 5.

14. Find the condition that the roots of $ax^3 + 3bx^2 + 3cx + d = 0$ may be in GP.

15. Find the unit tangent vector to the curve $x = t^2 + 1$, $y = 4t - 3$, $z = 2t^2 - 6t$ at the point $t = 2$.

16. The position vector of a particle in space at time t is $r(t) = e^{-t}i + 2\cos 3tj + 2\sin 3tk$. Find the velocity and acceleration vectors.

17. If $F = 6x^2zi + 2x^2yj - yz^2k$, find $div\,F$.

18. Find the work done by $F = xyi + yj - yzk$, over the curve $r(t) = ti + t^2j + tk$, $0 \le t \le 1$, in the direction of increasing t.

19. Integrate $f(x, y, z) = x^2 + z^2$ over the circle $(t) = a\cos tj + a\sin tk\ 0 \le t \le 2\pi$.

20. Show that curvature of a circle of radius a is $\frac{1}{a}$.

21. Find the length of the curve $r(t) = \sqrt{2}\,ti + \sqrt{2}tj + (1-t)k$ from $(0, 0, 1)$ to $(\sqrt{2}, \sqrt{2}, 0)$.

22. Establish the relation $\nabla \cdot (\nabla \times \bar{F}) = 0$.

(8 × 2 = 16 Marks)

PART – C

Answer any **six** questions. **Each** question carries **4** marks.

23. If $\alpha, \beta, \gamma, \delta$ are the roots of the equation $ax^4 + bx^3 + cx^2 + dx + e = 0$ such that $\alpha + \beta = \gamma + \delta$, show that $4abc - b^3 - 8a^2d = 0$.

24. Solve the equation $x^4 - 2x^3 - 21x^2 + 22x + 40 = 0$ whose roots are in AP.

25. Find the root of the equation $x^3 - 9x + 1 = 0$ correct to three decimal places using bisection method.

26. Describe the Newton-Raphson method of finding the solution of a general function $f(x) = 0$.

27. Establish the relation $\nabla \cdot (\nabla \times \overline{F}) = 0$.

28. Using Green's theorem calculate the integral $\oint_C y^2dx + x^2dy$ where C is the boundary of the triangle bounded by $x = 0$, $x + y = 1$, $y = 0$ in the counterclockwise direction.

29. Find the flux of $F = (x - y)i + xj$ across the circle $x^2 + y^2 = 1$ in the xy–plane.

30. Find the potential function for the field $(e^x \cos y + yz)i + (xz - e^x \sin y)j + (xy + z)k$.

31. Show that gradient field describing a motion is irrotational.

(6 × 4 = 24 Marks)

PART – D

Answer any **two** questions. **Each** question carries **15** marks.

32. (a) Solve the equation $4x^4 - 85x^3 + 357x^2 - 340x + 64 = 0$, given that roots are in geometric progression.

    (b) Solve the equation $15x^3 - 23x^2 + 9x - 1 = 0$ whose roots are in HP.

33. Use Newton-Raphson method to obtain a root, correct to three decimal places of $x - \cos x = 0$.

34. If $A = 2xyi + yz^2 j + xzk$ and S is a rectangular parallelepiped bounded by $x = 0, y = 0, z = 0$, $x = 2, y = 1, z = 3$ verify divergence theorem.

35. Verify Stoke's theorem, when $F = (2x - y)i - yz^2 j - y^2 zk$, S is the upper half surface of the unit sphere $x^2 + y^2 + z^2 = 1$ and C is its boundary.

(2 × 15 = 30 Marks)
