

Reg. No. :

Name :

Second Semester B.Sc. Degree Examination, May 2019

First Degree Programme under CBCSS

Complementary Course for Chemistry/Polymer Chemistry

MM 1231.2 : Mathematics II

INTEGRATION, DIFFERENTIAL EQUATIONS AND ANALYTIC GEOMETRY

(2014-2017 Admns)

Time : 3 Hours

Max. Marks : 80

SECTION – I

All the first ten question are compulsory, They carry 1 mark each.

1. Evaluate $\int \sqrt{x^2} dx$.
2. Find the average value of the function $f(x) = \sin x$ over the interval $[0, \pi]$.
3. A particle moves with acceleration $a(t) = -2 m/s^2$ and has velocity $V_0 = 3 m/s$ at time $t = 0$. Find the displacement of the particle during the time interval $1 \leq t \leq 4$.
4. Solve $y' = \frac{xy}{2}$.
5. Check whether the differential equation $(x^3 + 3xy^2)dx + (3x^2y + y^3)dy$ is exact? Justify your answer.
6. Find a general solution of $4y'' + 4y' - 3y = 0$.

7. Define conic section.
8. What is the latus rectum of the ellipse $3x^2 + 4y^2 = 48$?
- What is the reflection property of a parabola?
10. Obtain the semi focal axis and semi conjugate axis of the hyperbola $9x^2 - 16y^2 = 144$.

SECTION – II

(10 × 1 = 10 Marks)

Answer any eight questions from among the questions 11 to 22. These questions carry 2 marks each.

11. A projectile is fired vertically upwards with an initial velocity of 49 m/s from a tower of 150 m high. How long will it take the projectile to hit the ground? What will be its speed at impact?
12. Find the area of the region enclosed by the curves $x = \sin y$, $x = 0$, $y = \pi/4$ and $y = 3\pi/4$?
13. Derive the formula for the volume of a sphere of radius r .
14. Find the arc length of the curve $x = \cos 2t$, $y = \sin 2t$, $0 \leq t \leq \pi/2$.
15. Solve $y' = x^2 - 2x + 2$.
16. Solve $xyy' = x^2 + y^2$.
17. Obtain the general solution of the equation $y'' - 6y' + 25y = 0$.
18. Obtain the particular integral of the equation $y' - 5y = 3e^x - 2x + 1$.
19. Find the equation of the ellipse whose minor axis is 4 and the distance between the foci is 2.

20. A hyperbola has for its asymptotes the straight lines $2x - y - 3 = 0$ and $3x + y - 7 = 0$ and passes through the point $(1, 1)$. Find its equation.
21. Find the equations to the directrices of the ellipse $2x^2 + 3y^2 = 6$.
22. Find the condition that the line $lx + my = 1$ is a normal to the parabola $y^2 = 4ax$.

(8 × 2 = 16 Marks)

SECTION – III

Answer any six questions from among the questions 23 to 31. These questions carry 4 marks each.

23. Evaluate the iterated integral $\int_0^1 \int_0^{\sqrt{1-x^2}} (x^2 + y^2) dy dx$ by converting to polar coordinates.
24. Find the area of the surface generated by revolving the parametric curve $x = t, y = 2t^2, 0 \leq t \leq 1$, about the y -axis.
25. Evaluate the triple integral $\iiint_G xy^2 \sin z dV$ where G is the set of points satisfying $-1 \leq x \leq 1, 0 \leq y \leq 1, 0 \leq z \leq \pi/2$.
26. Solve $(2x + 5y + 1) dx - (5x + 2y - 1) dy = 0$.
27. Obtain the general solution of $y' + y \tan x = y^3 \sec x$.
28. Solve $y'' - y' - 2y = \sin 2x$.
29. Find the vertex, focus, equation of the axis and directrix of the parabola $(y + 3)^2 = 2(x + 2)$.
30. Describe the graph of the equation $9x^2 - 16y^2 + 18x + 32y - 151 = 0$.
31. Find the locus of the point of intersection of perpendicular tangents to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$.

(6 × 4 = 24 Marks)

SECTION – IV

Answer any two questions from among the questions 32 to 35. These questions carry 15 marks each.

32. (a) Find the area in the first quadrant which is inside the circle $r = 4 \cos \theta$ and outside the Lemniscate $r^2 = 8 \cos 2\theta$.
- (b) Use cylindrical coordinates to find the volume of the solid enclosed by the paraboloid $z = x^2 + y^2$ and the plane $z = 9$.
33. (a) Find the orthogonal trajectory of coaxial circles $x^2 + y^2 + 2gx + c = 0$ where g is the parameter.
- (b) Solve $\frac{d^3y}{dx^3} + 2\frac{d^2y}{dx^2} + \frac{dy}{dx} = e^{2x} + \sin 2x$.
34. (a) Prove that the line tangent to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ at the point (x_1, y_1) has the equation $\frac{xx_1}{a^2} + \frac{yy_1}{b^2} = 1$.
- (b) Find the equation to the hyperbola with foci $(5, 2)$, $(-3, 2)$ and eccentricity 2.
35. (a) A planet has eccentricity $e = 0.249$ and semi major axis $a = 39.5$ AU.
- (i) Find equation of its orbit in polar coordinate system
- (ii) Find the period T in years
- (iii) Find the perihelion and aphelion distances.
- (b) Find the eccentricity and distance from the pole to the directrix and sketch the graph of $r = \frac{6}{2 + \cos \theta}$.

(2 × 15 = 30 Marks)