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Reg. No. : ....

Name : ....

Second Semester B.Sc. Degree Examination, December 2021

First Degree Programme Under CBCSS

Mathematics

Complementary Course for Chemistry/Polymer Chemistry

MM 1231.2 – Mathematics – II

CALCULUS WITH APPLICATIONS IN CHEMISTRY – II

(2020 Admission Regular)

Time : 3 Hours Max. Marks : 80

SECTION – I

All the first **ten** questions are compulsory. They carry **1** mark each.

1. Find the curl of a vector field $\bar{r} = x\hat{i} + y\hat{j} + z\hat{k}$.

2. Give an example of a conditionally convergent series.

3. Find the sum of the integers from 1 to 222.

4. Define divergence of a vector field.

5. State Taylors theorem for two variable functions.

6. Evaluate $\int_0^1 \int_0^1 xydxdy$.

7. Find f_{xy} of the function $f(x,y) = e^x \sin y$.

8. If $x = r\cos\theta$ and $y = r\sin\theta$, find $\frac{\partial(x,y)}{\partial(r,\theta)}$.

9. Write Maclaurin series of $(1-x)^{-1}$.

10. Find the gradient of a scalar field $\phi(x,y,z) = (x+y+z)$.

(10 × 1 = 10 Marks)

SECTION – II

Answer **any eight** questions from among the questions **11** to **26**.

These questions carry **2** marks each.

11. Find the sum of the series $\frac{\sin\theta}{1!} + \frac{\sin 2\theta}{2!} + \frac{\sin 3\theta}{3!} + \ldots$

12. Find the total differential of the function $f(x,y) = x^2 + y^2 - 2xy$.

13. By changing the order of integration, evaluate $\int_0^1 \int_0^{1-y} (x^2 + y^2)\, dx dy$.

14. Show that $\nabla.(\nabla\varphi \times \nabla\psi) = 0$, where φ and ψ are scalar fields.

15. Show that $(1 + 4xy + 2y^2)\, dx + (1 + 4xy + 2x^2)\, dy$ is exact.

16. Sum the series $S = 1 + \frac{3}{4} + \frac{5}{4^2} + \frac{7}{4^3} + \ldots$

17. Show that the differential $df = x^2 dy - (y^2 + xy)\, dx$ is not exact

18. Expand $f(x) = \sin x$ as Maclaurin series.

19. Find the Laplacian of a scalar field $\phi(x, y, z) = x^2y^2z^2$.

20. Express x, y and z in cylindrical polar coordinates.

21. Prove that $div(grad\ \phi) = \nabla^2\phi$.

22. Find the interval where the series $1 - x + x^2 - x^3 + ...$ is convergent. Also find the sum of the series when $x = \frac{1}{2}$.

23. Evaluate $\int_0^a\int_0^b\int_0^c (xyz)\, dxdydz$.

24. Prove that $\sum_{n=1}^{\infty}\frac{1}{n^3 + 2n^2 + 3n}$ is convergent.

25. Define stationary points.

26. Let $f(x, y) = (x + y)^2$, $x = \cos u$ and $y = \sin u$. Find $\frac{df}{du}$.

(8 × 2 = 16 Marks)

SECTION – III

Answer **any six** questions from among the questions **27** to **38**.

These questions carry **4** marks each.

27. Find the volume of the region bounded by the three coordinate surfaces $x = 0, y = 0$ and $z = 0$ and the plane $x + y + z = 1$.

28. Let $\bar{F}(x, y, z) = f_1(x, y, z)\,\hat{i} + f_2(x, y, z)\,\hat{j} + f_3(x, y, z)\,\hat{k}$ and let $\bar{F}$ has continuous second order partial derivatives. Show that $\nabla.(\nabla.\bar{F}) = 0$.

29. Find the divergence and curl of the vector field $\bar{r} = (x^2yz)\,\hat{i} + (xy^2z)\,\hat{j} + (xyz^2)\,\hat{k}$.

30. Evaluate $\int_{x=-1}^{1}\int_{y=0}^{\pi}\int_{z=0}^{1} x^2 \sin y \, e^z dzdydx$.

31. Let $f(x, y, z) = y^2 - 4ax + z^5$, $x = at^2$, $y = 2at$ and $z = \sin t$. Using chain rule to find $\frac{df}{dt}$.

32. Given that the series $\sum_{n=1}^{\infty} \frac{1}{n}$ diverges, determine whether the series $\sum_{n=1}^{\infty} \frac{10n^3 + 11}{n^4 - n}$ converges.

33. Evaluate $\int_{0}^{\infty}\int_{y=x}^{\infty} \frac{e^{-y}}{y} dx\, dy$ by changing the order of integration.

34. Evaluate the sum $\sum_{n=1}^{N} \frac{1}{n(n+1)(n+2)}$.

35. Find the Maclaurin series of $S(x) = \frac{1}{1+x^2}$. Using this expansion to find the Maclaurin series of $\tan^{-1} x$.

36. Evaluate the integral $\int_{0}^{\infty} e^{-x^2} dx$.

37. Given that $x(u) = 1 + au$ and $y(u) = bu^3$, find the rate of change of $f(x, y) = xe^{-y}$ with respect to u.

38. Evaluate $\int_{x=0}^{1}\int_{y=x}^{1}\int_{z=0}^{y-x} dzdydx$.

(6 × 4 = 24 Marks)

SECTION – IV

Answer **any two** questions from among the questions **39** to **44**.

These questions carry **15** marks each.

39. Find the Taylor expansion of $f(x, y) = \sin x \sin y$ about the point $(x, y) = (0, 0)$ up to quadratic terms (second degree terms).

40. Find the stationary points of $f(x, y, z) = x^3 + y^3 + z^3$ subject to the following constraints $g(x, y, z) = x^2 + y^2 + z^2 = 1$ and $h(x, y, z) = x + y + z = 0$.

41. Evaluate the volume integral $\int_0^3 \int_0^4 \int_{x=y/2}^{x=(y/2)+1} \left(\frac{2x - y}{2} + \frac{z}{3} \right) dxdydz$.

42. Find the sum S_N of the first N terms of the following series, and hence determine whether the series are convergent , divergent or oscillatory

    (a) $\sum_{n=1}^{\infty} ln\left(\frac{n+1}{n}\right)$

    (b) $\sum_{n=1}^{\infty} (-2)^n$

    (c) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1} n}{3n}$.

43. Prove that

    $\nabla(\phi + \psi) = \nabla\phi + \nabla\psi$

    $\nabla.(a + b) = \nabla.a + \nabla.b$

    $\nabla \times (a + b) = \nabla \times a + \nabla \times b$

$$\nabla(\phi\psi) = \phi\nabla\psi + \psi\nabla\phi$$

$$\nabla.(\phi a) = \phi\nabla.a + a.\nabla\phi$$

$$\nabla.(a \times b) = b.(\nabla \times a) - a.(\nabla \times b)$$

$$\nabla \times (\phi a) = \nabla\phi \times a + \phi\nabla \times a.$$

44. Find the total surface area and volume of the sphere $x^2 + y^2 + z^2 = a^2$.

(2 × 15 = 30 Marks)
