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Reg. No. : ....

Name : ....

Second Semester B.Sc. Degree Examination, December 2021.

First Degree Programme under CBCSS

Mathematics

Complementary Course for Chemistry and Polymer Chemistry

MM 1231.2 MATHEMATICS II – CALCULUS WITH APPLICATIONS IN CHEMISTRY – II

(2018 – 2019 Admission)

Time : 3 Hours Max. Marks : 80

SECTION – I

All **ten** questions are compulsory, each caries **1** mark:

1. Find f_{yx} of the function $f(x,y) = xe^y + ye^x$.

2. If $w = f(x)$ be differentiable functions of x and $x = \varphi(t)$ be differentiable function of t then write the chain rule formula to find $\frac{dw}{dt}$.

3. Find the sum of even number between 1 and 101.

4. Find the formula to calculate S_N for a Arithmetico-geometric series.

5. Show that $\sum_{n=1}^{\infty}(n^2+2)$ is not convergent.

6. If $\vec{r}(t) = 5t^3\hat{i} + (3t+2)\hat{j} + 3t^2\hat{k}$ then $\frac{d\vec{r}}{dt} =$

7. If $F(x,y,z) = (x^2+y)\hat{i} + (y^2+z)\hat{j} + (x^2+z)\hat{k}$. Find $DivF$ at (1,1,2)

P.T.O.

8. If $\varphi(x, y, z) = xyz$, find $grad\varphi$ at (1,1,1).

9. Find $\int_0^a \int_0^b \int_0^c 8xyz\, dx\, dy\, dz$.

10. Suppose $f(x.y) = x + y$ defined over R in xy- plane given by $0 \le x, y \le 1$, then find average of f over the region R.

(10 × 1 = 10 Marks)

SECTION – II

Answer **any eight** questions, each carries **2** marks.

11. If $f(x, y) = x^2 + y^2 + 2xy$ then find $x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y}$.

12. If $f(x, y) = x^2y^2$ the find the total derivative df.

13. Given that $x(u) = 1 + au$ and $y(u) = bu^3$, where a and b are constant. Find rate of change of $f(x, y) = xe^{-y}$ with respect u.

14. Show that $\sum_{n=1}^{\infty} \frac{1}{n!}$ converges.

15. Determine the convergence of $\sum_{n=1}^{\infty} \frac{1}{(n!)^2}$.

16. Explain Cauchy's root test for the convergence of a series.

17. If $\varphi(x, y, z) = x^2 + y^2 + z^2$ then find $div(grad\varphi)$ at (1,1,-1).

18. Find Laplacian of $\varphi(x, y, z) = z + e^x \cos y$.

19. Show that $F(x, y, z) = (e^z + y)\hat{i} + (e^x - y)\hat{j} + (e^y + z)\hat{k}$ is solenoidal.

20. Evaluate $\int_0^1 \int_0^2 xy(x - y)dx\, dy$.

21. Evaluate $\int_0^1 \int_0^x \int_0^y xyz\, dx\, dy\, dz$.

22. Find the area enclosed between $x = 5$, $x = 10$ $y = x$ and $y = 5 + x$.

(8 × 2 = 16 Marks)

SECTION – III

Answer **any six** questions, each carries **4** marks

23. Find Taylor's theorem to find a quadratic approximation of $f(x,y) = xe^{y}$ about the origin.

24. The temperature of a point (x,y) on a unit circle is given by $T(x,y) = 1 + xy$ find the temperature of the two hottest point on the circle.

25. Sum the series $s(\theta) = 1 + \cos\theta + \frac{\cos 2\theta}{2!} + \frac{\cos 3\theta}{3!} + \ldots.$

26. Given that the series $\sum_{n=1}^{\infty} \frac{1}{n}$ diverge, determine whether the series converges: $\sum_{n=1}^{\infty} \frac{4n^2 - n - 3}{n^3 + 2n}$.

27. Show that $\nabla.(\nabla\varphi) = \nabla^2\varphi$ for any scalar field φ and verify for $\varphi(x,y,z) = e^{xyz}$.

28. If $F(x,y,z) = xyz\hat{i} + (2x+z)\hat{j} + (x+2y)\hat{k}$, then find $\nabla.(\nabla \times F)$.

29. Find unit tangent vector $\hat{t}$ and acceleration $\vec{a}$ of a particle moving along the trajectory $\vec{r}(t) = 5\cos t\hat{i} + 5\sin t\hat{j} + 3t\hat{k}$.

30. Find the centre of mass of the solid hemisphere bounded by the surfaces $x^2 + y^2 + z^2 = a^2$ and *xy plane* assuming that it has a uniform density ρ.

31. Find the moment of inertia of a uniform rectangular lamina of mass M with sides a *and b* about of the side of length *b*.

(6 × 4 = 24 Marks)

SECTION – IV

Answer **any two** questions, each carries **15** mark each

32. Find the stationary points of $f(x,y,z) = x^3 + y^3 + z^3$ subject to the following conditions

    (a) $g(x,y,z) = x^2 + y^2 + z^2 = 1$

    (b) $g(x,y,z) = x^2 + y^2 + z^2 = 1$ and $h(x,y,z) = x + y + z = 0$.

33. Determine the convergence of the following series

(a) $\sum_{n=1}^{\infty} \frac{2}{n^2}$

(b) $\sum_{n=1}^{\infty} \frac{n^n}{n!}$

(c) $\sum_{n=1}^{\infty} (-1)^n \frac{1}{n}$.

34. (a) Find the volume of tetrahedron bounded by the coordinate surfaces $x=0, y=0, z=0$ and the plane $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$.

(b) Find $\int_0^4 \int_{x=\frac{y}{2}}^{\frac{y}{2}+1} \frac{2x-y}{2} dx\, dy$ by applying the transformation $u=\frac{2x-y}{2}$ and $v=\frac{y}{2}$.

35. (a) Find the length of one turn of a helix $\vec{r}(t)=5\cos t\hat{i}+5\sin t\hat{j}+3t\hat{k}$; $0 \le t \le 2\pi$.

(b) Show that the acceleration of a particle travelling along a trajectory $\vec{r}(t)$ is given by $\vec{a}(t)=\frac{dv}{dt}\hat{t}+\frac{v^2}{\rho}\hat{n}$, where $\hat{t}$ is the unit tangent vector, v is the speed $\hat{n}$ is the principal normal vector and ρ is the radius of convergence.

(2 × 15 = 30 Marks)