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Reg. No. : ....

Name : ....

Third Semester B.Sc. Degree Examination, February 2024

First Degree Programme under CBCSS

Mathematics

Complementary Course for Chemistry and Polymer Chemistry

MM 1331.2 : MATHEMATICS III – LINEAR ALGEBRA, PROBABILITY THEORY AND NUMERICAL METHODS

(2018–2020 Admission)

Time : 3 Hours Max. Marks : 80

PART – A

All the first **ten** questions are compulsory and each carries **1** mark.

1. Find the rank of the matrix $\begin{bmatrix} 1 & 2 \\ 2 & 4 \\ 3 & 7 \end{bmatrix}$.

2. Check whether the matrix is $\begin{bmatrix} 3 & 1 & 2 \\ 0 & 4 & 1 \\ 3 & 2 & 2 \end{bmatrix}$ singular.

3. Find the row echelon form of the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 4 & 1 \\ 4 & 6 & 4 \end{bmatrix}$.

4. Define Hermitian matrix.

5. If $P(A) = 0.8$, $P(B) = 0.5$ and $P(A \cap B) = 0.32$ find $P(A \cup B)$.

P.T.O.

6. What is the formula for the probability mass function of a binomial distribution?

7. Explain the concept of mutually exclusive events.

8. At a party, there are 2 types of chips (potato and tortilla) and 3 types of dips (salsa, guacamole, and cheese). Define the sample space for selecting one chip and one dip.

9. Define trapezium rule.

10. Write the iterative formula for finding the approximate value of $\sqrt{5}$ using Newton Raphson method.

(10 × 1 = 10 Marks)

PART – B

Answer any **eight** questions. **These** questions carry **2** marks.

11. When do we say that a matrix is in row reduced echelon form.

12. Find the equation of a line passing through (1,1,1) and perpendicular to the plane $x - y - z = -6$.

13. If 1 and 2 are the eigen value of the matrix $\begin{bmatrix} 2 & 3 \\ x & y \end{bmatrix}$, find the values of x and y.

14. Find the rank of the matrix $\begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & -4 \\ 0 & 4 & 0 \end{bmatrix}$.

15. Find the probability of getting exactly two heads when tossing a fair coin three times.

16. The random variable X has the density $f(x) = \begin{cases} kx^2\,(10 - x), & 0 \le x \le 1 \\ 0, & elsewhere \end{cases}$. Find the value of k.

17. Find the coefficient of x^4 in the binomial expansion of $(1+x)^{12}$.

18. If a fair six-sided die is rolled, what is the probability of getting either an even number or a multiple of 3?

19. The mean and variance of a binomial random variable X are 16 and 8 respectively. Find $P(X=0)$ and $P(X=1)$.

20. Evaluate $\int_0^1 \frac{1}{1+x^2}\,dx$ using Simpson's method with step size $h = 0.25$.

21. Explain binary chopping.

22. What are the equations for the Adams-Moulton Predictor-Corrector method.

(8 × 2 = 16 Marks)

PART – C

Answer any **six** questions. **These** question carry **4** marks each.

23. Find the eigen value and eigen vectors of the matrix $\begin{bmatrix} 1 & 1 & 2 \\ -1 & 2 & 1 \\ 0 & 1 & 3 \end{bmatrix}$.

24. (a) Find unit vector perpendicular to both $a = i - 2j + 3k$ and $b = i + 2j - k$.

(b) Find the angle between the vectors $a = i - 2j + 3k$ and $b = 3i - 2j + k$.

25. Find the row reduced echelon form of the matrix $\begin{bmatrix} 1 & 1 & 3 \\ 4 & 1 & 3 \\ 1 & 1 & 1 \\ 2 & 4 & 5 \end{bmatrix}$ and determine its rank.

26. A fair die is rolled. What is the probability of an even number turning up? If the die is not fair such that faces 1 to 5 are equally likely while face 6 is twice as likely as any other face, what would be the probability of an even number turning up?

27. There are 250 typographical errors in a book of 1000 pages. The number of errors per page is supposed to follow a Poison distribution. What is the probability a randomly selected page will have more than 2 errors?

28. Find the number r such that the area under the normal distribution curve $y = f(x)$ from $\mu - r$ to $\mu + r$ is equal to $\frac{1}{4}$.

29. Out of 1000 randomly chosen families with 4 children each, determine the expected number of families that will have (a) at least one boy, (b) 1 or 2 girls, and (c) no girls.

30. Solve the system of equations

$27x + 6y - z = 85$

$x + y + 54z = 110$

$6x + 15y + 2z = 72$

Using Gauss-Seidel method

31. Evaluate $\int_2^4 \frac{1}{\log_{10} x} dx$ using (a) trapezoidal rule and (b) Simpson's rule, using step size $h = 0.25$.

(6 × 4 = 24 Marks)

PART – D

Answer any **two** questions. **These** questions carries **15** marks each

32. Find the value of $y(0.2)$ by fourth order Runge-Kutta method, given $\frac{dy}{dx} = e^x + y$ with $y(0) = 0$.

33. Solve the initial value problem $y' = -2xy^2$, $y(0) = 1$ for y at $x = 1$ with step length 0.2 using Taylor series method of order four.

34. X is a normal random variable with mean 50 and standard deviation 10.

    (a) Find $P(X \leq 48)$

    (b) Find $P(40 < X < 55)$

    (c) Find the value of α and β such that $P(X < \alpha) = 0.1$ and $P(X > \beta) = 0.05$

35. Find the matrix of transformation that diagonalize the matrix $\begin{bmatrix} 3 & 1 & -1 \\ -2 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix}$ Also, find the diagonal matrix.

(2 × 15 = 30 Marks)
