VTM NSS COLLEGE, DHANUVACHAPURAM

DEPARTMENT OF MATHEMATICS

QUESTION BANK : SEMESTER 3 (MATHS FOR PHYSICS)

LINEAR ALGEBRA,SPECIAL FUNCTIONS AND CALCULUS

2 MARKS

1. Show that if A is a square matrix (i)A+A' is Symmetric
   a. A-A' is Skew Symmetric
2. Find the sum and product of eigen values of the matrix $\begin{bmatrix} 2 & 0 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$
3. If A and B are matrices such that A+B=$\begin{bmatrix} 1 & -1 \\ 3 & 0 \end{bmatrix}$ and A-B=$\begin{bmatrix} 3 & 1 \\ 1 & 4 \end{bmatrix}$.Find A and B.
4. Find the rank of the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 2 \\ 2 & 6 & 5 \end{bmatrix}$
5. State Cayley-Hamilton theorem and find the characteristic equation of $\begin{bmatrix} 2 & 1 \\ 3 & 5 \end{bmatrix}$.
6. Find the eigen value of the matrix $\begin{bmatrix} 2 & 0 \\ 0 & -2 \end{bmatrix}$.
7. Show that for any square matrix A, A and A' have the same eigen values.
8. If A=$\begin{pmatrix} 1 & 4 \\ 2 & 3 \end{pmatrix}$, show that $A^2 - 4A - 5I = 0$.
9. Find the rank of the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 2 \\ 2 & 6 & 5 \end{bmatrix}$
10. Find the sum and product of eigen values of the matrix $\begin{bmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{bmatrix}$
11. Find the rank of the matrix $\begin{bmatrix} 2 & 4 & 6 \\ 4 & 8 & 12 \end{bmatrix}$.
12. Evaluate the determinant $\begin{vmatrix} 1 & -5 & 2 \\ 7 & 3 & 4 \\ 2 & 1 & 5 \end{vmatrix}$.
13. Find the rank of the matix $\begin{bmatrix} 2 & -3 & 5 & 3 \\ 4 & -1 & 1 & 1 \\ 3 & -2 & 3 & 4 \end{bmatrix}$.
14. If A and B are matrices such that A+B=$\begin{bmatrix} 1 & -1 \\ 3 & 0 \end{bmatrix}$ and A-B=$\begin{bmatrix} 3 & 1 \\ 1 & 4 \end{bmatrix}$,find A and B.
15. Show that x=a Cos*nt* is a solution of the differential equation $\frac{d^2x}{dt^2}$+n^2x=0
16. Solve y'=-y/x,given y(1)=1
17. Solve $\frac{dy}{dx} = e^{3x-2y} + x^2e^{-2y}$

18. Write the order and degree of the differential equation $(\frac{dy}{dx})^3 + 2y = (\frac{d^2y}{dx^2})^2$
19. Find an integrating factor of the differential equation $(x+1)\frac{dy}{dx} - y = e^{3x}(x+1)^2$
20. Check whether the differential equation (y cosx +1)dx + sinx dy = 0 is exact
21. Solve $\frac{d^4y}{dx^4} + 13\frac{d^2y}{dx^2} + 36y = 0$
22. Write the general form of Cauchy's homogeneous linear equation
23. Find the Wronskian of e^x and e^{-x}
24. Solve $\frac{dy}{dx} = e^{3x-2y} + x^2e^{-2y}$
25. Transform the differential equation $x^2\frac{d^2y}{dx^2} + 3x\frac{dy}{dx} + y = \frac{1}{(1-x)^2}$ into linear equation with constant coefficients
26. Find the divergence of $\nabla = xyz\, I + 3x^2y\, j + (xz^2 - y^2z)k$ at the point (2,-1,1)
27. Evaluate $\int_c (1 + xy^2)\, ds$ where c : r(t) = ti+2tj , $0 \le t \le 1$
28. Find $\iint_\sigma (x^2 + y^2 + z^2)ds$, where σ is the sphere of radius 2 centred at the origin
29. Using divergence theorem , find the outward flux of the vector field F(x,y,z) = zk across the sphere $x^2+y^2 + z^2= a^2$
30. Use divergence theorem to find the outward flux of the vector field $F(x,y,z)=2xi+3yj+z^2k$
31. State stokes theorem
32. Verify that $y = e^{-3x}$ is a solution of y''+y'-6y = 0
33. Find the integrating factor of $y'-y = e^{2x}$
34. Verify whether the equation $xydx+(2x^2+3y^2-20)dy=0$ is exact or not.
35. Using Green's theorem evaluate $\int 4xydx + 2xydy$ where C is the rectangle bounded by x=-2,x=4,y=1,y=2.
36. Solve y''-5y'+6y = 0
37. State the recurrence relation for gamma function
38. Prove that the force field $F=ie^y+j\, xe^y$ is conservative in the entire xy-plane
39. State Guass's law for inverse square field
40. What is the outward flux of the vector field F = xi+yj+zk , across any unit cube.

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41. Find the eigen values and eigen vectors of the matrices $\begin{bmatrix} 4 & 3 \\ 2 & 9 \end{bmatrix}, \begin{bmatrix} 1 & 4 \\ 3 & 2 \end{bmatrix}$
42. Find the eigen values and eigen vectors of the matrices (i)$\begin{bmatrix} 4 & 3 \\ 2 & 9 \end{bmatrix}; \begin{bmatrix} 1 & 4 \\ 3 & 2 \end{bmatrix}$
43. Find the inverse of the matrix $\begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{bmatrix}$.
44. Find x,y,z and w given that $3\begin{bmatrix} x & y \\ z & w \end{bmatrix} = \begin{bmatrix} x & 5 \\ -1 & 2w \end{bmatrix} + \begin{bmatrix} 6 & x+y \\ z+w & 5 \end{bmatrix}$.

45. Show that the matrix $\begin{bmatrix} 1/3 & -2/3 & 2/3 \\ 2/3 & -1/3 & -2/3 \\ 2/3 & -2/3 & 1/3 \end{bmatrix}$ is orthogonal.

46. Find the eigen values and eigen vectors of the matrix $\begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$.

47. Verify Cayley- Hamilton theorem for the matrix $A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$ and find its inverse.

48. Evaluate the determinant D= $\begin{vmatrix} 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{vmatrix}$

49. Find the eigen values and eigen vectors of the matrix A= $\begin{bmatrix} 1 & -2 \\ -5 & 4 \end{bmatrix}$.

50. Find the characteristic equation of the matrix A= $\begin{bmatrix} 1 & -1 & 1 \\ 4 & 1 & 0 \\ 8 & 1 & 1 \end{bmatrix}$. Hence find A^{-1}.

51. If A= $\begin{bmatrix} cos\alpha & sin\alpha \\ -sin\alpha & cos\alpha \end{bmatrix}$, find A^2 and hence find A^n.

52. Using Cramer's rule solve the set of equations:

$$2x + 3y = 3$$
$$x - 2y = 5$$

53. Write and row reduce the augmented matrix for the equations:

$$x - y + 4z = 5$$
$$2x - 3y + 8z = 4$$
$$x - 2y + 4z = 9.$$

54. Using Cayley Hamilton theorem evaluate A^{-1}, given A= $\begin{bmatrix} 4 & 6 & 6 \\ 1 & 3 & 2 \\ -1 & -4 & -3 \end{bmatrix}$

55. Find the number of solutions of the following system of equations 2x+6y+11=0 , 6x+20y-6z+3=0, 6y-18z+1=0.

56. Find the divergence and curl of the vector field F(x,y,z) = $x^2yi+2y^3zj+3zk$

57. Evaluate the surface integral $\iint_{\sigma} x^2 dS$ over the sphere $x^2+y^2+z^2 = 1$

58. Evaluate the surface integral $\iint_{\sigma} y^2z^2 dS$ where σ is the part of the cone $z = \sqrt{x^2+y^2}$ that lies between the planes z =1 and z = 2

59. Evaluate the line integral $I = \oint_C xdy$, where C is the circle in the xy plane defined by $x^2+y^2 = a^2$, z= 0

60. Use a line integral to find the area enclosed by the ellipse $x^2 / a^2 + y^2 / b^2= 1$

61. Evaluate $\int_c (3x^2+y^2)dx + 2xy\, dy$ along the circular arc C given by x = cost, y = sint $(0\leq t \leq \pi/2)$

62. Find the workdone by the force F = xi + 2yj , when it moves a particle on the curve $2y = x^2$ from (0,0) to (1,1)

63. Use divergence theorem to evaluate $\iint F.n\, ds$ where $F = (x^2-yz)i+(y^2-xz)j+(z^2-yz)k$ taken over the region bounded by x = 0, x = a, y = 0, y = b, z= 0 , z = c

64. Use green's theorem to evaluate $\int x^2ydx + xdy$ where C is the triangle with vertices (0,0) , (1,0) and (1,2)

65. Show that $\Gamma(\frac{1}{2}) = \sqrt{\pi}$

66. Solve $x\frac{dy}{dx}+y=xy^3$

67. Show that $\beta(p,q)=\frac{\Gamma(p)\Gamma(q)}{\Gamma(p+q)}$

68. Show that $\beta(m,n) = \beta(n,m)$

69. Verify Green's theorem for $\int(xy+y^2)dx + x^2dy$ where C is closed,the curve consisting of the line y=x and the parabola $y=x^2$

70. Prove curl $(\varphi\overline{F}) = \varphi\, curl(\overline{F}) + \nabla\varphi \times \overline{F}$

71. Find the work done by the conservative field $\overline{F}(x,y) = e^y$i+xe^yj on a particle that move from (1,0) to (-1,0) along a semicircular path.

72. Apply Green's theorem to evaluate $\int_c (2x^2-y^2)\, dx + (x^2+y^2)dy$ where C is the boundary of the area enclosed by the x-axis and the upper half of the circle $x^2+y^2 = a^2$

73. Solve $1+yx\frac{dx}{dy}+x^2 = 0$ using variable separable method.

74. Find the general and singular solutions of y = px+a/p

75. Solve $1+yx\frac{dx}{dy}+x^2 = 0$

76. Solve $(y''+2y'+3)^2 = 0$

77. Find the orthogonal trajectories of the family of co-axial circles $x^2+y^2+2\lambda x + c = 0$ where λ is the parameter

78. Solve $y = 2px-p^3$

79. Using the method of variation of parameters solve $\frac{d^2y}{dx^2} + 4y = \tan 2x$

80. Solve $[\, x \tan(\frac{y}{x}) - y \sec^2(\frac{y}{x})\,]dx - x \sec^2(\frac{y}{x})dy = 0$

15 MARKS

81. Verify Stoke's theorem for the vector field F(x,y,z)=2zi+3xj+5yk, taking the surface σ to be the portion of the paraboloid Z= 4 - x^2-y^2 for which z≥0 upward orientation and C to be the positively oriented circle $x^2+y^2 = 4$ that forms the boundary of σ in the xy plane.
82. (a) Find the area of the surface extending upward from the circle $x^2+y^2 = 1$ in the xy-plane to the parabolic cylinder $z = 1-x^2$

(b) Suppose that a semicircular wire has the equation y=$\sqrt{25 - x^2}$ and that the mass density is $\delta(x, y)$ = 15-y.The density of the wire decreases linearly with respect to y to a value of 10 units at the top(y=5).Find the mass of the wire.

83. (a) Evaluate the surface integral $\iint_\sigma \ xz\, dS$ where σ is the part of the plane x+y+z = 1 that lies in the first octant

(b)Suppose that a curved lamina σ with constant density $\delta(x, y, z)$ = δ_0 is the portion of the paraboloid $z = x^2+y^2$ below the plane z = 1.Find the mass of the lamina

84. (a) Evaluate $\iint F.n\, ds$ where F =4xi-2y²j+z²k taken over the cylindrical region bounded by $x^2+y^2 = 4$, z = 0 ,z = 3

(b) Verify green's theorem for f(x,y) = y^2-7y, g(x,y) = 2xy + 2x and C is the circle $x^2+y^2 = 1$

85. Find the matrix P which transforms the matrix A=$\begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$ to the diagonal form. Hence calculate A^4.
86. (a) Find for what values of a and b, the equations

$$x + y + z = 6$$
$$x + 2y + 3z = 10$$
$$x + 2y + az = b \text{ have}$$

(i) No solution
(ii) a unique solution
(iii) more than one solution

(b) Find the values of k for which the equations

$3x + y - kz = 0$
$4x - 2y - 3z = 0$
$2kx + 4y + kz = 0$ may possess non-trivial solution.

87. Diagonalize the symmetric matrix A=$\begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}$.
88. Reduce the matxix A=$\begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$ to normal form and hence find the rank.
89. Find the eigen values and eigen vectors of the matrix M=$\begin{bmatrix} 1 & -4 & 2 \\ -4 & 1 & -2 \\ 2 & -2 & -2 \end{bmatrix}$.
90. Investigate the value of λ and μ so that the equations $2x + 3y + 5y = 9, 7x + 3y - 2z = 8, 2x + 3y + \lambda z = \mu$ have

(a) No solution (b) a unique solution (c) an infinite number of solutions.

91. (a) Find the value of a and b for which the equations

$$x + ay + z = 3$$
$$x + 2y + 2z = b$$
$$x + 5y + 3z = 9$$

Is (i) consistent and have unique solution

(ii) is inconsistent

(iii) is consistent and have infinitely many solutions

(b) Find the value of k for which

$$(3k - 8)x + 3y + 3z = 0$$
$$3x + (3k - 8) + 3z = 0$$
$$3x + 3y + (3k - 8) = 0$$

have a non-trivial solution.

92. (a) Solve $x\frac{dy}{dx} + y = x^4y^4$ (b) Solve $\frac{dy}{dx} = \frac{x+2y-3}{2x+y-3}$

93. Solve $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = e^{-x}$

94. Show that the equation $(2xy+y - \tan y)dx + (x^2 - x\tan^2 y + \sec^2 y + 2)\,dy = 0$ is exact and hence solve it.

95. Solve $(1-x^2)\frac{d^2y}{dx^2} - 3x\frac{dy}{dx} - y = 1$

96. (a)Find the orthogonal trajectory of the cardiods $r = a(1-\cos\theta)$

(b)Solve $(D-2)^2y = 8(e^{2x} + \sin 2x + x^2)$

97. (a)Solve $(3y+2x+4)dx - (4x+6y+5)dy = 0$

(b) Solve $(xy^3+y)dx + 2(x^2y^2+x+y^4)dy = 0$

98. Use the variation of parameters method to solve $\frac{d^2y}{dx^2} + y = \text{cosec}\, x$ subject to the boundary conditions $y(0) = y(\pi/2) = 0$

99. Solve by the method of undetermined coefficients $\frac{d^2y}{dx^2} - y = e^{3x}\cos 2x - e^{2x}\sin 3x$

100. (a) Solve $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = e^x$

(b) Solve $(y-x)\frac{dy}{dx} + 2x + 3y = 0$