VTM NSS COLLEGE, DHANUVACHAPURAM

DEPARTMENT OF MATHEMATICS

QUESTION BANK: SEMESTER 1 CORE MATHEMATICS 2023 ONWARDS

2 MARKS

(Logic and Proof)

1. Whether the following statement is true or false:
   (i)"If 3 is odd or 4>6 then 9≤5".
   (ii) 5 is not prime or 8 is prime
2. Write the negation of the statement : "If the sequence (a_n) is monotone and bounded, then (an) is convergent.
3. If the function $g(n,m) = n^2 + n + m$ where n and m are positive integers, then find g(16,17)
4. Let f be a function given by $f(x) = 4x + 7$. Use the contra-positive implication to prove the statement: "If $x_1 \neq x_2$ then $f(x_1) \neq f(x_2)$".
5. Is $x^2+3x-2=0$ a statement? If not, rewrite it as a statement.
6. Define the sentential connective ∇ and find the truth table for $(p\nabla p)\nabla(q\nabla q)$.

p	q	$p\nabla q$
T	T	F
T	F	F
F	T	F
F	F	T

7. Write the negation of the statement : "there exist x>2 such that f(x)=7".
8. Define conjunction
9. What is a bi-conditional statement
10. Define contradiction
11. Give an example of a tautology
12. Find the antecedent and consequent in the following statement
    (i) You can work here only if you have a college degree
    (ii) If n is an integer, then 2n is an even integer
13. Prove that |x|≥0 ∀x
14. Rewrite the statement "There exist a number less than 7 using $\exists, \forall$ *and* $\ni$ as appropriate.
15. Write the truth table for pVq
16. Provide a counter example to the statement "Every continuous function is differentiable".
17. Write the contrapositive statement of the statement "Continuity is a necessary condition for differentiability".

(Number theory)

18. Using recursion, evaluate (18,30,60,75,132)
19. Prove that 1+2+3+….+n = $\frac{n(n+1)}{2}$
20. Express (28,12) as a linear combination of 28 and 12
21. Prove that every integer $n \geq 2$ has a prime factor
22. Show that $n^3 - n$ is divisible by 2

23. Prove that there is no positive integer between 0 and 1

(Methods of Differential calculus)

24. Find all critical points of $3x^{\frac{5}{3}} - 15x^{\frac{2}{3}}$
25. Verify Rolle's theorem for $f(x) = x^2 - 5x + 4$ in the interval (1,4).
26. What is the velocity interpretation of the mean value theorem?
27. State Pappus theorem
28. Evaluate $\lim_{x\to\infty} \sqrt{x^8 + 7} - x^4$
29. Evaluate $\lim_{x\to\pi/4} (1 - tanx)sec2x$
30. Evaluate $\lim_{x\to 0} sinx^{tanx}$
31. Suppose that a particle moves with velocity $v(t) = \cos(\pi t)$ along a coordinate line. Assuming that the particle has coordinate s=4 at time t=0, find its position at time t.
32. Find the horizontal and vertical tangents of the curve $y = 6x^{\frac{1}{3}} + 3x^{\frac{4}{3}}$
33. The diameter of a sphere is measured with percentage error within $\pm 0.4\%$. Estimate the percentage error in the calculated volume of the sphere.

<u>4 MARKS</u>

(Logic and Proof)

34. Construct the truth table for the statement $[(\sim q) \wedge (p => q) => (\sim p)$
35. Prove or give a counter example that "for every integer n, n^2+3n+8 is even.
36. Prove that "If 7m is an odd number, then m is an odd number.
37. Which of the following statements are true? Justify?
    (i) If $m^2>0$, then m>0
    (ii) If m>0, then $m^2>0$
38. Use the truth table to verify that $p => q \ and \ \sim q =>\sim p$ are logically equivalent
39. Prove that "If x is a real number, then x≤|x|"
40. Prove that "If the sum of a real number with itself is equal to its square, then the number is 0 or 2.
41. Prove that "If x is a real number and if x>0, then (1/x)>0".

(Number theory)

42. Prove that there are infinitely many primes
43. Let a and b be positive integers. Then prove that [a,b]=ab/(a,b). Using the canonical decomposition of 18 and 24. Find their LCM.
44. State Inclusion-Exclusion Principle. Find the number of positive integers in the range 1976 through 3776 that are divisible by 13 or 15.
45. Show that the number of leap years after l 1600 and not exceeding a given year Y is given by $l = \left\lfloor \frac{y}{4} \right\rfloor - \left\lfloor \frac{y}{100} \right\rfloor + \left\lfloor \frac{y}{400} \right\rfloor - 388$
46. State and prove the Principle of Mathematical Induction
47. Prove that every non empty set of non negative integers has a least element
48. Prove that there is no polynomial f(n) with integral coefficients that will produce primes for all integers n
49. Find the number of positive integers $\leq$ 2076 and divisible by neither 4 nor 5
50. Prove that gcd of positive integers a and b is a linear combination of a and b
51. Find the canonical decomposition of 2520
52. Using recursion evaluate [24,28,36,40]

53. Prove that there are infinitely many primes of the form (4n+3)
54. Prove that every integer n $\geq$ 2 has a prime factor
55. Prove that every composite number n has a prime factor $\leq \ \lfloor\sqrt{n}\rfloor$
56. Prove that every person in a set of n people is of the same sex
57. Let b be an integer ≥2 .Suppose b+1 integers are randomly selected. Prove that the difference of two of them is divisible by b
58. Prove that any postage of n (≥ 2) cents can be made with 2- and 3-cent stamps

(Methods of Differential calculus)

59. Find the absolute maximum and minimum values of the function $f(x) = 2x^3 - 15x^2 + 36x$ on [1,5].
60. Find the intervals on which (i) $f(x) = x^2 - 4x + 3$ (ii) $f(x) = x^3$is increasing or decreasing.
61. Evaluate $\lim\limits_{x\to\infty} \frac{x^{\frac{-4}{3}}}{\sin\frac{1}{x}}$ & $\lim\limits_{x\to\frac{\pi}{2}} \frac{1-sinx}{cosx}$
62. Assume that oil spilled from a ruptured tanker spreads in a circular pattern whose radius increases at a constant rate of 2ft/s. How fast is the area of the spill increasing when the radius of the spill is 60ft.
63. Verify Mean Value Theorem for the function $f(x) = x^3 - 3x^2 + 2x$ in [0 , ½]
64. Suppose the side of a square is measured with a ruler to be 10 inches with a measurement error of at most ±1/32 inch. Estimate the error in the computed area of the square.
65. Suppose that a particle moves along a coordinate lines so that its velocity at time t is $v(t) = 2 + cost$. Find the average velocity of the particle during the time interval [0,∏]
66. Determine whether the function $f(x) = \frac{1}{x^2-x}$ has any absolute extrema on the interval (0,1). If so, find them.
67. Find the absolute extrema if any of the function $f(x0 = e^{(x^3-3x^2)}$ on the interval (0,+∞)
68. Verify Rolles theorem for $f(x) = x^2 - 8x + 15$ on [3,5]
69. Use an appropriate linear approximate to estimate the value of $\sqrt{24}$
70. Find the dimension of the rectangle with maximum area that can be inscribed in a circle of radius 10cm.
71. Explain steps for solving applied maximum and minimum problems.
72. Find dy/dx if $y = \frac{x^2-1}{x^3}$
73. State sufficient condition for f(x) to be concave up and concave down.

<u>15 MARKS</u>

(Logic and Proof)

74. Using the truth table show that the statement $[p \wedge \quad \sim q] => [p => q]$ is a tautology.
75. Write the four different types of negation statement of $\forall \varepsilon > 0, \ni N \in \mathcal{N}: if\ n \geq N, then\ \forall x\ in\ S, |f_n(x) - f(x)| < \varepsilon$
76. (a)Write the negation of the statement :
    (i)“If the sequence (an) is convergent, then (an) is monotone and bounded.
    (ii)M is a cyclic subgroup.
    (b)Construct a truth table for the compound statement ~(p⋀q)=>[(~p)V(~q)]
77. Determine the truth value of the statement $\forall x, \exists y \to x + y = 3$. Justify
78. Write the truth table of $p \vee (q \wedge r) \qquad and \qquad (p \vee q) => \sim r$

(Number theory)

79. Show that "If p and p^2+2 are primes, then p^3+2 is also a prime.
80. State and prove second principle of mathematical induction
81. State and prove division algorithm
82. State and prove Euclid theorem
83. (a) Let a and b be any positive integers, and r the remainder, when a is divided by b. Then prove that (a,b) = (b,r)
    (b)Using (a) evaluate (2076,1776)

(Methods of Differential calculus)

84. Let $f(x) = x^3 - 3x^2 + 1$. Determine the intervals on which f is increasing, decreasing, concave up and concave down. Locate all inflection points of f.
85. A camera mounted at a point 3000 ft from the base of a rocket launching pad. If the rocket is rising vertically at 880ft/sec, when it is 4000ft above the launching pad, how fast must the camera elevation angle change at that instant to keep the camera aimed at the rocket.
86. (i) An open box is to be made from a 16 inch by 30-inch piece of cardboard by cutting out squares of equal size from the four corners and binding p sides. What size should the squares be to obtain a box with largest volume.
    (iii) Prove that has no absolute maximum
87. Let $f(x) = x^3 - 3x^2 + 1$. Determine the intervals on which f is increasing, decreasing, concave up and concave down. Locate all inflection point of f. Also draw a rough sketch of the graph of f.
88. (i) A garden is to be laid out in a rectangular area and protected by a chicken wire fence. Which is the largest possible area of the garden if only 100 running feet of chicken wire is available for the fence.
    (ii) State and prove Mean Value Theorem
89. A golfer makes a successful chip shot to the green. Suppose that the path of the ball from the moment it is struck to the moment it hits the green is described by $y = 12.54x - 0.41x^2$ where x is the horizontal distance in yards from the point where the ball is struck and y is the vertical distance moment it is struck to the moment it hits the green. Assume that the fairway and green are at the same level.
90. Find a point on the curve $y = x^2$ that is closest to the point (18,0).
91. Evaluate $\lim_{x\to 0}(1 + sinx)^{\frac{1}{x}}$ using L'Hospital Rule.
92. Find the radius and height of the right circular cylinder of largest volume that can be inscribed in a right circular cone with radius 6 inches and height 10 inches.
93. Use Logarithmic differentiation to find $\frac{d}{dx}(x^2 + 1)^{sinx}$,$\frac{d}{dx}(x^3 - 2x)^{lnx}$
94. Find $\frac{dy}{dx}$ $if\ y = \sin^{-1}(x^3)$, $y = x^2\,(\sin^{-1} x)^3$
95. Sketch the graph of $y = e^{-\frac{x^2}{2}}$ and identify the location of all relative extrema and inflection ponts.
96. What is the smallest possible slope for a tangent to the graph of the equation $y = x^3 - 3x^2 + 5x$
97. The path of a fly whose equation of motion are $x = \frac{cost}{2+sint}$, $y = 3 + sin2t - 2sin^2t$, $0 \leq t \leq 2\pi$. How high and low does it fly.
98. A liquid form of antibiotic manufactured by a pharmaceutical firm is sold in bulk at a price of $200 per unit. If the total production cost in dollars for x unit is $C(x) = 500000 + 80x +$

$0.003x^2$ and if the production capacity of the firm is at most 30000 units in a specified time, how many units of antibiotic must be manufactured and sold in that time to maximize the profit.

99. Find $\frac{dy}{dx}$ if $(i) y = \ln\left(\frac{x^2 sinx}{\sqrt{1+x}}\right)$ $(ii) y = \frac{x^2 \sqrt[3]{7x-4}}{(1+x^2)^4}$

100. Consider the function $f(x) = x^5 + x + 1$

(i) Show that f is one-to-one on the interval (-∞,+∞)

(ii) Find a formula for the derivative of f^{-1}

(iii) Compute $(f^{-1})'|(1)$.

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