V T M N S S COLLEGE DHANUVACHAPURAM

First Semester B.Sc. Degree Mathematics Question Bank

Complementary Course for Physics

MM 1131.1 : Calculus and Sequence and Series

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2 Mark

1. Evaluate $\lim_{x\to\infty}\left(\sqrt{x^6+6}-x^3\right)$

2. Show that the function f defined by $f(x)=\sqrt{4-x^2}$ is continuous on the closed interval (-2,2).

3. Find the derivative of $f(x)=\frac{2x^2+x}{x^3-1}$.

4. Find the derivative of $f(x)=In\sqrt{x^2+1}$.

5. Compute $\frac{ds}{dt}$if $s=(1+t)\sqrt{t}$.

6. Estimate $\frac{dy}{dx}$ if $y=\cos(x^3)$.

7. Find $\frac{d}{dx}[\ln(x^2+1]$.

8. Find the average rate of change of $y=x^2+1$ with respect to x over the interval [3,5].

9. Find $\frac{dy}{dx}$if $y=sin^{-1}(x^3)$.

10. Use implicit differentiation to find $\frac{d^2y}{dx^2}$if $4x^2-2y^2=9$.

11. Obtain the value of $\lim_{n\to\pi/2}\frac{1-sinx}{\cos x}$.

12. Evaluate $\lim_{x\to0^+}xlnx$

13. Evaluate $\int\frac{3x^2}{x^3+5}dx$.

14. Evaluate $\int_0^2 x(x^2+1)^3dx$

15. Evaluate $\int\frac{Cosx}{sin^2x}dx$.

16. Evaluate $\int \frac{dx}{1+3x^2}$.

17. Evaluate $\int cos^2 xdx$.

18. Evaluate $\int xe^x dx$

19. Find the area under the curve $f(x) = x^3$ over the interval [2,3].

20. Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$if $z = x^4 \sin(xy^3)$.

21. Define level surface for a function f(x,y,z). Describe the level surfaces of $f(x, y, z) = x^2 + y^2 + z^2$.

22. Describe the level surfaces of $f(x, y, z) = z^2 - x^2 - y^2$.

23. If $f(x, y) = x^2y^3 - x^4y$,find $\frac{\partial^2 f}{\partial y^2}$.

24. Find the local linear approximation to $f(x, y) = \sqrt{x^2 + y^2}$ at (3,4).

25. Express $\frac{\partial w}{\partial r}$ and $\frac{\partial w}{\partial s}$ in terms of r and s if $w = x + 2y + z^2, x = r/s, y = r^2 + lns, z = 2r$.

26. State the chain rules for derivatives.

27. Let $f(x) = \sqrt{1 - x^2 - y^2 - z^2}$. Find $f\left(0, \frac{1}{2}, \frac{1}{2}\right)$.

28. Consider the sphere $x^2 + y^2 + z^2 = 1$. Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ at the point $\left(\frac{2}{3}, \frac{1}{3}, \frac{2}{3}\right)$.

29. Let $f(x, y) = xsin(xy)$. Then find fx(x,y).

30. Given that $z = e^{xy}$, x=2u+v, y=u/v, compute $\frac{\partial z}{\partial u}$ and $\frac{\partial z}{\partial v}$.

31. State the ratio test.

32. Determine whether the sequence $\left\{\frac{n}{2n+1}\right\}_{n=1}^{\infty}$ converges or diverges by examining the limit as $n \to \infty$.

33. Find the sum of the series $\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$.

34. Use the alternating series test to check the convergence of $\sum_{k=1}^{\infty} (-1)^{k+1} \frac{1}{k}$.

35. Using the root test cheke the convergence of the series $\sum_{k=2}^{\infty}\left(\frac{4k-5}{2k+1}\right)^k$.

36. Find the Maclaurin series for e^x.

37. Define the Taylor series for f about x=x0.

38. Find all values of x for which the series $\sum_{k=0}^{\infty} x^k$converges and find the sum of the series for those values of x.

39. Determine whether the sequence $\left\{(-1)^{n+1}\frac{n}{2n+1}\right\}_{n=1}^{+\infty}$ or diverges

40. Evaluate$\sum_{k=0}^{\infty}\frac{5}{4^k}$.

4 Mark

1. Find $\lim_{x\to 1}\frac{x-1}{\sqrt{x}-1}$.
2. Find $\lim_{x\to -\infty}\frac{4x^2-x}{2x^3-5}$.
3. Find $\lim_{x\to +\infty}\frac{\sqrt{x^2+2}}{3x-6}$.
4. Compute $\lim_{x\to 5}\frac{x^2-3x-10}{x^2-10x+25}$.
5. Evaluate $\lim_{x\to 1}\frac{x^2+x-2}{x^2-x}$.
6. Estimate i) $\lim_{x\to +\infty}\frac{x}{e^x}$ ii) $\lim_{x\to 0+}\frac{\ln x}{\csc x}$.
7. Find $\lim_{x\to 0}(1+\sin x)^{1/x}$.
8. Show that |x| is continuous every where.
9. Find $\frac{dy}{dx}$if $y=\frac{sinx}{1+\cos x}$.
10. Find the equation of the tangent line to the curve $y=\frac{2}{x}$ at the point (2, 1) on the curve.
11. Find dy/dx if $y = sin(1 + cosx)$.
12. Evaluate $\frac{d}{dx}\left[sin\sqrt{1+cosx}\right]$.
13. Find the derivative of $y=\frac{x^2\sqrt{7x-14}}{(1+x^2)^4}$.
14. Evaluate $\int(x^2+x)dx$

15. Evaluate $\int xe^x dx$.

16. Evaluate $\int \left(\frac{1}{x^2} + sec^2\pi x\right) dx$.

17. Evaluate $\int_2^5 (2x-5)(x-3)^9 dx$.

18. Find

a) $\int_0^1 f(3x+1)\,dx$ if $\int_1^4 f(x)\,dx = 5$

b) $\int_2^0 xf(x^2)\,dx$ if $\int_0^4 f(x)\,dx = 1$

19. Evaluate $\int sin^4 x cos^5 x dx$.

20. Let $f(x,y) = y^2e^x + y$. Find f_{xyy}.

21. Find the second order partial derivatives of f(x,y) = x2y3 + x4y.

22. Suppose $w = \sqrt{x^2+y^2+z^2}$ x = cosθ, y = sinθ, z = tanθ. Find $dw/d\theta$ when $\theta = \pi/4$.

23. Use chain rule to find $\frac{\partial w}{\partial u}$ and $\frac{\partial w}{\partial v}$, where $w = e^{xyz}, x = 3u + v, y = 3u - v, z = u^2v$.

24. Use appropriate forms of the chain rule to find $\frac{\partial w}{\partial \rho}$ and $\frac{\partial w}{\partial \theta}$ where $w = x^2 + y^2 + z^2$, $x = \rho sin\emptyset cos\theta, y = \rho sin\emptyset sin\theta$ and $z = \rho cos\emptyset$.

25. Locate all relative extrema and saddle points of $f(x,y) = 4xy - x^4 - y^4$.

26. Find the second order partial derivatives of f(x,y) = x2y3 + x4y.

27. Suppose that w = x2 + y2 + z2, x = cosθ, y = sinθ, z = tanθ. Use the chain rule to find $dw/d\theta$ when $\theta = \pi$.

28. Locate all relative extrema and saddle points of $f(x,y) = 3x^2 - 2xy + y^2 - 8y$.

29. Let L(x,y) denote the local linear approximation to $\sqrt{x^2+y^2}$ at the point (3,4). Compare the error in approximating $f(3.04,3.98) = \sqrt{(3.04)^2 + (3.98)^2}$ by L(3.04,3.98) with the distance between the points (3,4) and (3.04,3.98).

30. Show that the integral test applies and use the integral to determine whether the series $\sum_{k=1}^{\infty} \frac{1}{k^2}$ converge or diverge.

31. Use the comparison test to determine whether the series $\sum_{k=1}^{\infty} \frac{1}{2k^2+k}$ converge or diverge.

32. Find the sum of the series $\sum_{k=1}^{\infty} \left(\frac{3}{4^k} - \frac{2}{5^{k-1}}\right)$.

33. Test for the convergence of $\sum_{k=1}^{\infty} (-1)^{k+1} \frac{k+3}{k(k+1)}$.

34. Find the interval of convergence and radius of convergence of $\sum_{k=1}^{\infty} \frac{(x-5)^k}{k^2}$.

35. Use an nth Maclaurin polynomial for e^x to approximate e to five-decimal place accuracy.

36. Find the first four Taylor polynomials for ln x about x=2.

37. Find $\lim_{x\to+\infty} \sqrt{x^6+5-x^3}$.

38. Find the domain of the function

$$f(x) = \frac{1\cdot 3\cdot 5\cdots(2k-1)}{(2k-2)!}x^k.$$

39. Check for the convergence of the series

$$\sum_{k=1}^{\infty}(-1)^{k+1}\frac{k+3}{k(k+1)}.$$

40. Test the convergence of the following series

i) $\sum_{k=1}^{\infty}\frac{k^k}{k!}$ ii) $\sum_{k=1}^{\infty}\frac{1}{2^k-1}$

15 Mark

1. (a) Prove that $\lim_{x\to 3} x^2 = 9$.

   (b) Prove that $\lim_{x\to 81} \sqrt{x}$=9

2. (a) Find $\frac{dy}{dx}$if $y = 3x^8 - 2x^5 + 6x + 1$.

   (b) At what points, does the graph of $y = x^3 - 3x + 4$ have a horizondal tangent line?

   (c) Find the area of the triangle formed from ther coordinate axes and the tangent line to the curve $y = 5x^{-1} - \frac{1}{5}x$ at the point (5,0).

3. (a) Use implicit differentiation to find $\frac{d^2y}{dx^2}$if $4x^2 - 2y^2 = 9$.

   (b) Find the slopes of the tangent lines to the curve $y^2 - x + 1 = 0$ at the points (2,-1) and (2,1).

   (c) Find Find$f''\left(\frac{\pi}{4}\right)$if $f(x) = cosecx$.

4. Find

$$\int \frac{3x^4+4x^3+16x^2+20x+9}{(x+2)(x^2+3)^2}\,dx.$$

5. Evaluate $\int \frac{x^2+x-2}{3x^3-x^2+3x-1}\,dx$.

6. (a) Evaluate $\int sin^4\, x\, cos^4\, x\, dx$.

   (b) Evaluate $\int tan^2\, x\, sec^4\, x\, dx$.

7. (a) Let $f(x) = \begin{cases} -\frac{xy}{x^2+y^2}, & (x,y) \neq (0,0) \\ 0 & (x,y) = (0,0) \end{cases}$. Show that $f_x(x,y)$ and $f_y(x,y)$ exist at all points (x,y).

(b) Show that the function u(x,t)=sin(x-ct) is a solution of one dimentional wave equation.

8. a) Find the absolute maximum and minimum values of f(x,y) = 3xy−6x−3y+7.

(b)At what points on the circle $x^2 + y^2 = 1$ does f(x,y) = xy have an absolute maximum and what is that maximum?

9. (a) Find the n^{th} Maclaurin polynomial for $\frac{1}{1-x}$ and express it in sigma notation.

(b) Find the n^{th} Taylor polynomial for $\frac{1}{x}$ about $x = 1$ and express it in sigma notation.

10. Find the interval of convergence and radius of convergence of the following series

i) $\sum_{k=1}^{\infty} \frac{(x-5)^k}{k^2}$ ii)$\sum_{k=1}^{\infty} \frac{(x)^k}{k!}$

11. (a) Evaluate $\int_1^{\sqrt{2}} \frac{dx}{x^2\sqrt{4-x^2}}$.

(b) Find the slope of the circle $x^2 + y^2 = 25$ at the point (3,-4).

12. Use implicite differentiation to find $\frac{dy}{dx}$if $y^2 = x^2 + sinxy$.

(b) Find the tangent to the curve $x^3 + y^3 - 9xy = 0$ at the point $(2,4)$.

13. (a) Find the interval of convergence and radius of convergence of $\sum_{k=1}^{\infty} \frac{(x-5)^k}{k^2}$.

(b) Find the values of x for which the power series $\sum_{k=1}^{\infty} k!\, x^k$ converge.

(c) Find the values of x for which the power series $\sum_{k=1}^{\infty} \frac{(-1)^{k-1} x^{2k-1}}{2k-1}$ converge.

14. (a) Find $f''\left(\frac{\pi}{4}\right)$if $f(x) = secx$.

(b) On a sunny day, a 50ft flagpole casts shadow that changes with the angle of elevation of the sun. Let s be the length of the shadow and θ the angle of elevation of the sun. Find the rate at which the length of the shadow is changing with respect to θ when $\theta = 45^0$. Express your answer in units of feet/degree.

(c) Compute $\frac{d}{dx}\left[ln\left(\frac{x^2 \sin x}{\sqrt{1+x}}\right)\right]$.

15. (a) Find the slope of the sphere $x^2 + y^2 + z^2 = 1$ in the y-direction at the points $\left(\frac{2}{3}, \frac{1}{3}, \frac{2}{3}\right)$and $\left(\frac{2}{3}, \frac{1}{3}, \frac{-2}{3}\right)$.

(b) Describe the level surfaces of $f(x,y,z) = x^2 + y^2 + z^2$.

16. Use Lagrange multipliers to determine the dimentions of a rectangular box, open at the top, having a volume of 32 ft^3 and requiring the least amount of material for its construction.

17. (a) Use the comparison test to determine whether the following series converge or diverge

i) $\sum_{k=1}^{\infty} \frac{1}{\sqrt{k}-\frac{1}{2}}$ ii) $\sum_{k=1}^{\infty} \frac{1}{2k^2+k}$

(b) Prove that the series $\sum_{k=1}^{\infty} \frac{1}{k(k+1)}$ converges. Find the sum.

18. Find the interval of convergence of the series

a) $\sum_{k=0}^{\infty} \frac{(-1)^k x^k}{3^k (k+1)}$.

b) $\sum_{k=0}^{\infty} \frac{(-1)^k (x-4)^k}{(k+1)^2}$.

c) $\sum_{k=1}^{\infty} \frac{x^k}{k(k+1)}$.

19. Find the Maclaurin series for

(a) e^x (b) $\sin x$ (c) $\cos x$ (d) $\frac{1}{1-x}$

20. a) Evaluate $\frac{d}{dx} sec^{-1}(5x^4)$.

b) Find $\frac{\partial z}{\partial x}$ if the equation $yz - lnz = x + y$ defines z as a function of two independent variables x and y.
