

Reg. No. : .....

Name : .....

Third Semester B.Sc. Degree Examination, January 2023

First Degree Programme under CBCSS

Mathematics

Complementary Course for Chemistry and Polymer Chemistry

MM 1331.2 : MATHEMATICS III — LINEAR ALGEBRA, PROBABILITY  
THEORY AND NUMERICAL METHODS

(2018 Admission)

Time : 3 Hours

Max. Marks : 80

SECTION I

All the first 10 questions are compulsory. Each carries 1 mark.

1. Define the rank of a matrix.

2. Evaluate the determinant  $\begin{vmatrix} 0 & a & -b \\ -a & - & c \\ b & -c & 0 \end{vmatrix}$ .

3. What is the magnitude of a vector?

4. Define Kronecker  $\delta$ .

5. What is the scalar product of two vectors?

6. What is the sample space of an event?

7. There are 10 chairs in a row and 8 people to be seated. In how many ways can this be done?

P.T.O.

8. Write the expression for variance of a random variable  $x$  and explain the terms.
9. Write Baye's formula for conditional probability.
10. What is an algebraic equation?

(10 × 1 = 10 Marks)

### SECTION II

Answer **any eight** questions. Each carries **2** marks.

11. Find the rank of the matrix  $\begin{pmatrix} 1 & 1 & 2 \\ 2 & 4 & 6 \\ 3 & 2 & 5 \end{pmatrix}$ .

12. Evaluate the determinant  $\begin{vmatrix} 1 & -5 & 2 \\ 7 & 3 & 4 \\ 2 & 1 & 5 \end{vmatrix}$ .

13. Find the cross product of the vectors  $A = 2i + j - k$  and  $B = i + 3j - 2k$ .
14. Find the symmetric equation of the line through  $(1, -1, -5)$  and  $(2, -3, -3)$ .
15. Define linear functions.
16. Find the probability that a single card drawn from a shuffled deck of cards will be either a diamond or a king.
17. Two dice are rolled. What is the probability that the sum is  $\geq 10$ ?
18. Define mutually exclusive events.
19. What is the probability that a number  $n, 1 \leq n \leq 99$ , is divisible by both 6 and 10?
20. A club consists of 50 members. In how many ways can a president, vice president, secretary and treasurer be chosen?
21. Write Newton-Raphson iteration formula.
22. Evaluate the integral  $I = \int_0^1 \frac{1}{1+x^2} dx$  using the trapezium rule.

(8 × 2 = 16 Marks)

### SECTION III

Answer any six questions. Each carries 4 marks.

23. Write and row reduce the augmented matrix for the equations :

$$x - y + 4z = 5$$

$$2x - 3y + 8z = 4$$

$$x - 2y + 4z = 9$$

24. Using Cramer's rule solve the set of equations :

$$2x + 3y = 3$$

$$x - 2y = 5$$

25. Find the equation of a line through  $(1, 0, -2)$  and perpendicular to the plane  $3x - 4y + z + 6 = 0$ .

26. Two students are working separately on the same problem. If the first student has probability  $\frac{1}{2}$  of solving it and the second student has probability  $\frac{3}{4}$  of solving it, what is the probability that atleast one of them solves it?

27. Find the coefficient of  $x^8$  in the binomial expansion of  $(1+x)^{15}$ .

28. Using Baye's formula find the probability of all heads in three tosses of a coin if you know that atleast one is a head?

29. Evaluate  $I = \int_0^2 (x^2 - 3x + 4) dx$  using trapezium rule with  $h = 0.5$ .

30. Evaluate  $I = \int_0^1 \frac{1}{1+x^2} dx$  using Gaussian integration.

31. Find an explicit formula that will generate a random number  $y$  distributed on  $(-\infty, \infty)$  according to the Cauchy distribution  $f(y)dy = \left(\frac{a}{\pi}\right) \frac{dy}{a^2 + y^2}$ , given a random number  $\xi$  uniformly distributed on  $(0, 1)$ .

**(6 × 4 = 24 Marks)**

## SECTION IV

Answer any two questions from this section. Each carries 15 marks.

32. Diagonalize  $H = \begin{pmatrix} 2 & 3-i \\ 3+i & -1 \end{pmatrix}$ .

33. A preliminary test is customarily given to the students at the beginning of a certain course. The following data are accumulated after several years:

- (a) 95% of the students pass the course.
- (b) 96% of the students who pass the course also passed the preliminary test.
- (c) 25% of the students who fail the course passed the preliminary test.

What is the probability that a student who failed the preliminary test will pass the course?

34. Derive the Poisson density function  $P_n = \frac{\mu^n}{n!} e^{-\mu}$ .

35. Solve the simultaneous equations

$$x_1 + 6x_2 - 4x_3 = 8$$

$$3x_1 - 20x_2 + x_3 = 12$$

$$-x_1 + 3x_2 + 5x_3 = 3$$

Using Gaussian elimination.

(2 × 15 = 30 Marks)