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Reg. No. : ....

Name : ....

Third Semester B.Sc. Degree Examination, January 2023

First Degree Programme under CBCSS

Mathematics

Complementary Course for Chemistry and Polymer Chemistry

MM 1331.2 : MATHEMATICS III — LINEAR ALGEBRA, PROBABILITY THEORY AND NUMERICAL SOLUTIONS

(2021 Admission)

Time : 3 Hours Max. Marks : 80

SECTION – I

All the first **ten** questions are compulsory. They carry 1 mark each.

1. Give an example of a square matrix.

2. What is an elementary matrix?

3. Define a regular linear transformation.

4. Define eigen value of a matrix.

5. Find the number of permutations of all the letters of the word 'Committee'.

6. What is a random variable?

7. Write two properties of normal distribution.

P.T.O.

8. The iterative formula for finding the reciprocal of N is $x_{n+1} =$ ____________

9. Evaluate $\Delta \tan^{-1} x$.

10. State trapezoidal rule.

(10 × 1 = 10 Marks)

SECTION – II

Answer **any eight** questions. These questions carry **2** marks each.

11. Find the rank of the matrix $\begin{bmatrix} 2 & 4 & 6 \\ 4 & 8 & 12 \end{bmatrix}$.

12. Find the value of k for which the system of equations $(3k-8)x+3y+3z=0$, $3x+(3k-8)y+3z=0$, $3x+3y+(3k-8)z=0$ has a nontrivial solution.

13. State Cayley-Hamilton theorem and find the characteristic equation of $\begin{bmatrix} 2 & 1 \\ 3 & 5 \end{bmatrix}$.

14. Find the eigen valve of the matrix $\begin{bmatrix} 2 & 0 \\ 0 & -2 \end{bmatrix}$.

15. Show that for any square matrix A, A and A′ have the same eigen values.

16. What is the chance that a leap year selected at random will contain 53 Sundays?

17. Find the probability of getting a king of red colour from a well shuffled deck of 52 cards?

18. Evaluate $p(A/B)$ and $p(B/A)$ given $p(A)=1/4$ and $p(B)=1/3$.

19. In 256 sets of 12 tosses of a coin, in how many cases, one can expect 8 heads and 4 tails?

20. Use a binomial distribution to calculate $P(X=0)$ and $P(X=1)$.

21. Suppose 5 cards are drawn at random from a pack of 52 cards. If all cards are red, find the probability that all of them are hearts.

22. Find a real root of the equation $x^3-2x-5=0$ by the method of false position correct to three decimal places.

23. Evaluate $\sqrt{5}$ by Newton's iteration method.

24. Find the missing term in the table

x	2	3	4	5	6
y	45	49.2	54.1	–	67.4

25. Evaluate $\int_0^6 \frac{dx}{1+x^2}$ by using trapezoidal rule.

26. Find a solution using Simpson's 1/3 rule

x	0	0.1	0.2	0.3	0.4
$f(x)$	1	0.9975	0.9900	0.9776	0.8604

(8 × 2 = 16 Marks)

SECTION – III

Answer **any six** questions. These question carry 4 marks each.

27. Find the inverse of the matrix $\begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{bmatrix}$.

28. Find x, y, z and w given that $3\begin{bmatrix} x & y \\ z & w \end{bmatrix} = \begin{bmatrix} x & 5 \\ -1 & 2w \end{bmatrix} + \begin{bmatrix} 6 & x+y \\ z+w & 5 \end{bmatrix}$.

29. Show that the matrix $\begin{bmatrix} 1/3 & -2/3 & 2/3 \\ 2/3 & -1/3 & -2/3 \\ 2/3 & -2/3 & 1/3 \end{bmatrix}$ is orthogonal.

30. Find the eigen values and eigen vectors of the matrix $\begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$.

31. Verify Cayley – Hamilton theorem for the matrix $A=\begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$ and find its inverse.

32. Two cards are drawn in succession from a pack of 52 cards. Find the chance that the first is a king and the second queen if the first card is

    (a) replaced (b) not replaced.

33. Three identical boxes contain red and white balls. The first box contains 3 red and 2 white balls, the second box has 4 red and 5 white balls, and the third box has 2 red and 4 white balls. A box is chosen very randomly and a ball is drawn from it. If the ball that is drawn out is red, what will be the probability that the second box is chosen?

34. A die is tossed thrice. A success is "getting 1 or 6" on a toss. Find the mean and variance of the number of successes.

35. Find the cubic polynomial which takes the following values :

x	0	1	2	3
$f(x)$	1	2	1	10

    Hence evaluate $f(4)$.

36. If $y_{10}=3$, $y_{11}=6$, $y_{12}=11$, $y_{13}=18$, $y_{14}=27$, find y_4.

37. Use Trapezoidal rule to estimate the integral $\int_0^2 e^{x^2}\,dx$ taking 10 intervals.

38. Find $y(0.2)$ for $y'=x^2y-1$, $y(0)=1$ with step length 0.1 using Taylor series method.

(6 × 4 = 24 Marks)

SECTION – IV

Answer **any two** questions. These question carry 15 marks each.

39. Reduce the matrix $A=\begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$ to normal form and hence find the rank.

40. Investigate the value of λ and μ so that the equations $2x+3y+5z=9$, $7x+3y-2z=8$, $2x+3y+\lambda z=\mu$ have

    (a) No solution (b) a unique solution (c) an infinite number of solutions.

41. A biased coin is tossed till a head appears for the first time

    (a) What is the probability that the number of required tosses is odd.

    (b) Two persons A and B toss an unbiased coin alternatively on the understanding that the first who gets the head wins. if A starts the game, find their respective chance of winning.

42. A random variable X has the following probability function :

x	0	1	2	3	4	5	6	7
$p(x)$	0	k	2k	2k	3k	k^2	$2k^2$	$7k^2+k$

    (a) Find the value of k.

    (b) Evaluate

        (i) $P(X<6)$

        (ii) $P(X\geq 6)$ and

        (iii) $P(0<X<5)$.

43. Using Newton's iterative method, find the real root of the equation $3x = \cos x + 1$.

44. Apply Gauss-Jordan method to solve the equations

$x + y + z = 9,\ 2x - 3y + 4z = 13,\ 3x + 4y + 5z = 40$.

(2 × 15 = 30 Marks)
