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Reg. No. : ....

Name : ....

Third Semester B.Sc. Degree Examination, January 2023

First Degree Programme under CBCSS

Statistics

Complementary Course for Mathematics

ST 1331.1 – STATISTICAL DISTRIBUTIONS

(2019 Admission onwards)

Time : 3 Hours Max. Marks : 80

SECTION – A

Answer **all** questions. **one** word to maximum of **one** sentence.

1. If X follows the Binomial distribution $B(n, 0.35)$ then $Y = n - X$ follows ———— distribution.
2. If mean of the Poisson distribution is 3, its third central moment is ————.
3. Variance of Bernoulli distribution is ————.
4. Define Standard normal distribution.
5. The fourth central moment of standard normal distribution is ————.
6. If $F \sim F(m, n)$, What is the distribution of I/F?
7. Define Sampling distribution.
8. If X follows a standard normal distribution then X^2 follows ————.
9. Define F statistic.
10. What is the relation between t and F distribution?

(10 × 1 = 10 Marks)

P.T.O.

SECTION – B (Short Answer)

Answer **any eight** questions. Each question carries **2** marks.

11. If X and Y are independently distributed random variables such that $X \sim B(4, 0.5)$ and $Y \sim B(3, 0.5)$, then obtain the distribution of $X + Y$.

12. If X is distributed according to Poisson with mean 1.5. Find its variance and mode.

13. Find the mean of geometric distribution with p.m.f. $f(x) = q^x p,\ x = 0, 1, 2, \ldots$

14. Define Beta distribution first type. If X follows Beta distribution first type what is the distribution of $Y = 1 - X$?

15. If $X \sim N(22, 2)$ and $Y \sim N(18, 4)$ and are independent, then find the distribution of $X + Y$.

16. If the mgf of a random variance X is $M_X(t) = e^{6t+t^2}$, find the mean and variance of X.

17. Derive the mean and variance of uniform distribution over $(0, \theta)$.

18. Define convergence in probability.

19. State Lindberg Levy form of Central Limit Theorem.

20. State Bernoulli's law of large numbers.

21. If X is uniformly distributed with mean 1 and variance 4/3, find $P(X < 0)$.

22. X is a random variable with mean 5 and variance 3. Find the lower bound to $P\{|X - 5| < 3\}$.

23. How large a sample should be taken from a normal population with mean 10 and S.D. 3, if the sample mean is to lie between 8 and 12 with probability 0.95?

24. Obtain the moment generating function of Chi square distribution.

25. Distinguish between parameter and statistic.

26. Define student's t distribution.

(8 × 2 = 16 Marks)

SECTION – C (Short essay)

Answer **any six** questions . Each question carries **4** marks.

27. Obtain mode of Poisson distribution.

28. Derive the mean and variance of hypergeometric distribution.

29. If $X \sim U(0, 1)$, find the distribution of $Y = -2 \log X$. Identify the distribution.

30. State and prove the additive property of gamma distribution.

31. If X is an exponential distribution with pdf $f(x) = \theta e^{-\theta x}$, $x > 0$, $\theta > 0$, Obtain the m.g.f. of X and hence obtain the mean of the distribution.

32. Show that exponential distribution posses lack of memory property.

33. Show that the sum of independent exponential random variables follows Gamma distribution.

34. If X is a normal variate with mean 20 and S.D. 5. Find the probability that
    (a) $16 \leq x \leq 22$,
    (b) $x \geq 23$
    (c) $|x = 20| > 5$.

35. A random variable X takes the values –1, 1, 3, 5 with probabilities 1/6, 1/6, 1/6 and 1/2 respectively. Find by direct computation $P\{|X-3| \geq 1\}$. Also find an upper bound to this probability by applying Chebychev's inequality.

36. Examine whether the weak law of large numbers holds for the sequence $\{X_k\}$ of independent random variables defined as follows.

$$P(X_k = \pm 2^k) = 2^{-(2k+1)},\ P(X_k = 0) = -2^{-2k}$$

37. State and prove the additive property of Chi square distribution.

38. Establish the relation between normal, Chi-square, t and F distribution.

(6 × 4 = 24 Marks)

SECTION – D

Answer **any two** questions. Each question carries **15** marks.

39. Derive the recurrence formula for central moment of Binomial distribution and hence discuss its skewness.

40. Show, stating the conditions clearly, that Binomial distribution leads to Poisson distribution.

41. (a) Derive the moment generating function of standard normal distribution.

    (b) If $X \sim N(\mu, \sigma)$ distribution, find the quartile deviation of X and also obtain the ratio between quartile deviation and standard deviation.

42. (a) State and prove Chebychev's inequality.

    (b) If X follows a binomial distribution with $n = 100$ and $p = \frac{1}{2}$, obtain using Chebychev's inequality a lower limit for $P\{|X - 50| < 7.5\}$.

43. (a) Obtain the sampling distribution for the sample mean from a normal population.

    (b) A population is known to follow the normal distribution with mean 2 and standard deviation 3. Find the probability that the mean of a sample of size 16 taken from this population will be greater than 2.5

44. Fit a binomial distribution to the following data. Also find the theoretical frequencies,

x	:	0	1	2	3	4	Total
f	:	8	32	34	24	5	103

(2 × 15 = 30 Marks)