

Reg. No. : .....

Name : .....

Third Semester B.Sc. Degree Examination, March 2022

First Degree Programme under CBCSS

Mathematics

Complementary Course for Physics

MM 1331.1 – MATHEMATICS III – CALCULUS AND LINEAR ALGEBRA

(2018 Admission)

Time : 3 Hours

Max. Marks : 80

SECTION – I

All the ten questions are compulsory. They carry 1 mark each.

1. Define degree of an ODE.
2. What is the auxillary equation of  $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = e^x$ .
3. Write the general form of Bernoulli's equation.
4. Prove that  $\vec{a} = (xy^2 + z)\vec{i} + (x^2y + 2)\vec{j}$  is conservative.
5. State Stoke's theorem.
6. Find the average value of  $\sin x$  on  $(-\pi, \pi)$ .
7. What are the Fourier coefficients of an even function  $f(x)$  in the interval  $(-l, l)$ .

8. Evaluate  $\begin{vmatrix} 1 & -5 & 2 \\ 7 & 3 & 4 \\ 2 & 1 & 5 \end{vmatrix}$ .

9. Find the rank of the matrix  $\begin{pmatrix} 1 & 1 & 2 \\ 2 & 4 & 6 \\ 3 & 2 & 5 \end{pmatrix}$ .

10. Find the trace of the matrix  $\begin{pmatrix} 1 & 0 & -1 \\ 2 & -3 & 0 \\ 1 & -3 & 2 \end{pmatrix}$ .

SECTION – II

Answer **any eight** questions from among the questions **11** to **22**. These questions carry **2** marks each.

11. Solve  $x \frac{dy}{dx} + 3x + y = 0$ .

12. Solve  $y = px + p^2$

13. Find the complementary function of the equation  $\frac{d^2y}{dx^2} + 2 \frac{dy}{dx} + y = 4e^{-x}$ .

14. Evaluate the line integral  $I = \int \vec{a} \cdot d\vec{r}$  where  $\vec{a} = (x+y)\vec{i} + (y-x)\vec{j}$  along the parabola  $y^2 = x$  from  $(1, 1)$  to  $(4, 2)$  in the  $xy$ -plane.

15. Find the vector area of the surface of the hemisphere  $x^2 + y^2 + z^2 = a^2$ ,  $z \geq 0$ , by evaluating the line integral  $S = \frac{1}{2} \oint_C \vec{r} \times d\vec{r}$  around its perimeter.

16. Define even and odd functions and give examples.

17. What are the Dirichlet conditions for the existence of the Fourier series of a periodic function?

18. Define Fourier transform.

SECTION – IV

Answer **any two** questions from among the questions 32 to 35. These questions carry 15 marks each.

32. Use Green's function to solve  $\frac{d^2y}{dx^2} + y = \operatorname{cosec} x$  subject to the boundary conditions  $y(0) = y\left(\frac{\pi}{2}\right) = 2$ .

33. Given the vector field  $\vec{a} = y\vec{i} - x\vec{j} + z\vec{k}$ , verify Stoke's theorem for the hemispherical surface  $x^2 + y^2 + z^2 = a^2, z \geq 0$

34. Let  $f(x) = \begin{cases} 1 & 0 < x < \frac{1}{2} \\ 0 & \frac{1}{2} < x < 1 \end{cases}$

Find :

- (a) a Fourier sine series
  - (b) a Fourier cosine series
  - (c) a Fourier exponential series, whose period is 1
35. (a) Solve  $x - z = 5, -2x + 3y = 1, x - 3y + 2z = -10$  by the method of finding the inverse of the coefficient matrix.
- (b) Find out whether the given vectors are dependent or independent; if they are dependent find a linearly independent subset.
- $(1, -2, 3), (1, 1, 1), (-2, 1, -4), (3, 0, 5)$
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