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Reg. No. : ....

Name : ....

Fourth Semester B.Sc. Degree Examination, August 2022

First Degree Programme Under CBCSS

Mathematics

Complementary Course for Physics

MM 1431.1 : MATHEMATICS IV – COMPLEX ANALYSIS, FOURIER SERIES AND FOURIER TRANSFORMS

(2014-2017 Admission)

Time : 3 Hours Max. Marks : 80

SECTION – I

All the **first ten** questions are compulsory. They carry **1** mark each.

1. Find the conjugate of $\frac{2+i}{4i+(1+i)^2}$.

2. Write down the Cauchy – Riemann equation.

3. Find $\lim_{z\to 2}\frac{z^2-4}{z-2}$.

4. Evaluate $\int_0^{1+i} z^2\,dz$.

5. State Liouville's theorem.

6. Find the radius of convergence for the power series $\sum\frac{z^n}{n}$.

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7. Find all the zeros of the function $f(z)=\frac{(z+1)^2(iz+2^3)}{z+7}$.

8. Find the smallest positive period of cos$x$.

9. Give an example for an even function.

10. Find the Fourier coefficient a_0 for the function $f(x)=\pi - x$ in the interval $(0, \pi)$.

(10 × 1 = 10 Marks)

SECTION – II

Answer **any eight** questions. These question carries **2** marks each.

11. Find all the values of $(-i)^{\frac{1}{3}}$.

12. Verify Cauchy – Riemann equations for the functions $f(z)=z^3$.

13. Let $f(z)=z^2+3z$, find u and v calculate the value of f at $z=1+3i$.

14. Evaluate $\int_C \frac{e^z}{z} dz$, where C is the circle $|z|=1$.

15. Test whether the series $\Sigma_{n=0}^{\infty} \frac{(100+75i)^n}{n!}$ is convergent or divergent.

16. Find the Maclaurin series of the function $f(z)=\tan^{-1} z$.

17. Show that the series $\Sigma_{m=0}^{\infty} \frac{z^m+1}{m^2+\cosh m|z|}$ converges uniformly in the disc $|z| \le 1$.

18. Determine the location and type of singularities of the function $f(z)=\sin 3z - \cos 3z$, including those at infinity.

19. Define periodic function and give an example.

20. Write down the Euler formula for calculating the fourier coefficient.

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21. Find the Fourier coefficient a_0 for the function $f(x) = \frac{\pi - x}{2}$ in the interval $(0, 2\pi)$.

22. Find the Fourier transform of $e^{-|t|}$.

(8 × 2 = 16 Marks)

SECTION – III

Answer **any six** questions. These questions carry **4** marks each.

23. Prove that $u = 2x - x^3 + 3xy^2$ is harmonic and find its harmonic conjugate. Also find the corresponding analytic function.

24. Evaluate $\oint L_n(1-z)dz$, C is the boundary of the parallelogram with vertices $\pm i, \pm(1+i)$.

25. State Cauchy's integral formula and hence evaluate $\int_C \frac{e^z}{z^2+4}dz$, where C is the positively oriented circle $|z - i| = 2$.

26. Find the centre and radius of convergence of the power series $\sum_{n=1}^{\infty} \frac{1}{n(n+1)}\left(\frac{z}{4}\right)^{n+1}$.

27. Expand $\frac{1}{z^2 - 3z + 2}$ in Laurent's series valid in the region $1 < |z| < 2$.

28. Find the residue of $f(z) = \frac{z^4}{z^2 - iz + 2}$ at singular points.

29. Find the Fourier series of the function $f(x) = \begin{cases} 0 & if \quad -2 < x < -1 \\ k & if \quad -1 < x < 1 \\ 0 & if \quad 1 < x < 2 \end{cases}$.

30. Obtain the half range cosine series for $f(x) = x$ in $(0, \pi)$.

31. Find the Fourier cosine transform of $f(x) = e^{-ax}$, where $a > 0$.

(6 × 4 = 24 Marks)

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SECTION – IV

Answer **any two** questions. These questions carry **15** marks each.

32. (a) Let C denote the boundary of the square whose sides lie along the lines $x=\pm 2$ and $y=\pm 2$, where C is described in the positive sense. Evaluate

(i) $\int_C \frac{z\,dz}{2z+1}$

(ii) $\int_C \frac{\cos z}{z(z^2+8)}dz$.

(b) Find the Taylor's series of the function $f(z)=\frac{1}{(z+i)^2}$ with centre $z_0=i$ and also find the radius of convergence.

33. (a) Show that $\int_{-\infty}^{\infty} \frac{\cos sx}{K^2+x^2}dx=\frac{\pi}{k}e^{-ks},(s>0,k>0)$.

(b) Evaluate $\int_0^{2\pi} \frac{1+4\cos\theta}{17-8\cos\theta}d\theta$.

34. Find the two half range expansion of the function $f(x)=\begin{cases} \frac{2k}{L}x & \text{if } 0<x<\frac{L}{2} \\ \frac{2k}{L}(L-x) & \text{if } \frac{L}{2}<x<L \end{cases}$.

35. (a) Using Fourier integral show that (a) $\int_0^{\infty} \frac{1-\cos\pi\omega}{\omega}\sin x\omega\, d\omega=\begin{cases} \frac{\pi}{2} & \text{if } 0<x<\pi \\ 0 & \text{if } x>\pi \end{cases}$.

(b) Find the Fourier transform of e^{-ax^2} where $a>0$.

(2 × 15 = 30 Marks)

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