VTM NSS COLLEGE DHANUVACHAPURAM

DEPARTMENT OF MATHEMATICS

QUESTION BANK

4TH SEMESTER MATHEMATICS

ST 1431.1 STATISTICAL INFERENCE

2 Marks Questions

1. Explain Interval estimation
2. State Fisher Neyman factorization theorem
3. What are the properties of MLE?
4. If t is a consistent estimator of θ, show that t^2 is a consistent estimator of θ^2
5. Define sufficiency.Give an example of sufficient estimator of a given population
6. What are the sufficient conditions for a consistent estimator?
7. For the random sample $x_1, x_2, \ldots, x_n$ takes from a Poisson population with parameter λ obtain an unbiased estimator of $e^{-\lambda}$
8. If $f(x;\lambda)=\frac{e^{-\lambda}\lambda^{x}}{x\,!}, \lambda > 0$ for x=0,1,2,.. Then estimate λ by the method of moments.
9. Write down the (1-α)% confidence interval for the proportion of a binomial population.
10. Let $x_1, x_{2,} \ldots, x_n$ be the random sample from a population with pdf $f(x,\theta)=\theta x^{\theta-1}; 0<x<1; \theta>0$. Find the sufficient estimator of θ
11. What is the Method of moments?

12. Define MLE
13. Write two properties of moment estimator
14. What is the confidence coefficient?
15. Define consistency
16. Write the confidence interval for population variance.
17. Give an statistic which is consistent but not unbiased.
18. Write down the procedure for finding critical region.
19. Define type 1 error & type 2 error
20. Distinguish between simple & composite hypothesis.Give examples
21. State Neyman pearson lemma.
22. Define the most powerful test.
23. Find the MLE of the parameter θ based on a random sample taken from a population with pdf f(x)=1/θ;$0\leq x \leq \theta$
24. What is the power of a test?
25. Define significance level of a test
26. Define critical region
27. Define test statistic
28. What is p value?
29. Find the test statistic for testing the mean of a population for a large sample when the population SD σ is unknown.
30. What is local control?
31. What do you mean by one tailed test & two tailed test?
32. Give the test statistic in the case of the smallest test to test the equality of means of two normal populations (a) when the population SD s are known (b) When the population SD are unknown.

33. If λ follows a distribution with pdf f(x)=$\frac{1}{\theta}e^{x/\theta} x \geq 0, \theta > 0$ .To test $H_0: \theta = 5$ against H_1:10.H_0 is rejected if x≥ 15 .Obtain the probability of type 1 error.
34. Explain paired t test
35. Define standard error
36. Give the statistic under the null hypothesis of testing the mean of a population has a specified value for a large sample.
37. Write down the format for ANOVA table for two way ANOVA
38. What are the principles of experimental design?
39. What is the total sum of squares and how it is decomposed?
40. What are the assumptions underlying ANOVA?

4 Marks Questions

1. Find the sufficient statistics for p of binomial population.
2. Derive confidence interval for the difference of means of Normal population.
3. Find the MLE for the parameter involved in poisson distribution.
4. Obtain the moment estimator of μ & σ based on n observations from N(μ, σ^2).
5. In sampling from a normal population, examine whether the sample variance is an unbiased estimator of the population variance.
6. Obtain the MLE of λ, based on a random sample taken from poisson population with parameter λ

7. A random sample of size 15 from a normal population give $\overline{x} = 3.2$ & $s^2 = 4.24$ Determine the 90% confidence limits for σ^2.
8. Given that the frequency function f(x,θ)=$\frac{e^{-\theta}\theta^{x}}{x!}$ where x can assume only non negative integral values and given the following observed values 6,11,4,8,7,9,12,5,7. Find the MLE for θ.
9. X is uniformly distributed in (a,b). A sample of size 5 consists of the observation 3.1,0.2,1.6,5.2 &2.1 find estimates of a & b b the method of moments.
10. A private phone received 2 & 3 wrong calls in two randomly selected days. Assuming a poisson distribution, obtain a point estimate of the expected number of wrong calls in 4 days.
11. Find the power & significance level for the following : if f(x)=1/θ;0<x<θ the critical region is 0.5<x<1;H_0:θ=1 against H_1:θ=2.
12. Give the procedure of hypothesis.
13. What is a statistical test? Is it possible to say whether a hypothesis is right or wrong using a statistical test?
14. Outline Neyman pearson method of testing hypothesis .
15. Obtain the best critical region for the population f(x,θ)=θ $x^{\theta-1}$ $0 \leq x \leq 1$of size α for testing H_0:θ=$\theta_0$against H_1: θ= θ_1
16. Show that the best critical region for the hypothesis H_0: $\mu = \mu_0$ concerning the mean μ of a population distribution

against the alternative H_1:$\mu = \mu_1$ is of the form $\bar{x} \leq A$ if $\mu_0 > \mu_1$ & $\bar{x} \geq B$ if $\mu_1 > \mu_0$

17. Suppose random sample of size n is taken from the poisson population with pdf f(x)=$\frac{e^{-\lambda}\lambda^x}{x!}$; x=0,1,2,3,.. Give the most powerful critical region of size α for testing $\lambda = \lambda_0$ against $\lambda = \lambda_1(\lambda_1>\lambda_0)$

18. If X≥ 1 is the critical region for testing θ=2 against alternative θ=1 on the basis of a single observation from the population with pdf f(x)=$\theta e^{-\theta x}$;$0 \leq x \leq \theta$ obtain type 1error& type 2 error

19. A population follows the normal distribution with parameters μ &σ.To test the hypothesis H_0:$\mu = 5$ against H_1:$\mu = 7$ the test procedure suggested is to reject H_0 if $\bar{x} \geq 6$ where $\bar{x}$ is the sample mean of size 16. Find the significance level &power of the test.

20. If $x_1, x_2, \ldots, x_n$ are n independent observations on a random variable x with pdf f(x)=$\theta x^{\theta-1}$;0<x<1,θ≥ 1Show that the best critical region for testing H_0:$\theta = 1$against H_1:$\theta = 2$ can be defined in terms of the GM of $x_1, x_2, \ldots, x_n$

21. From a population with unknown SD a sample of size 100 was taken & its mean & SD were found to be 195 &50. Examine whether the hypothesis that the men of the population is 200 may be justified a 5% level of significance.

22. A form of Intelligence test was given to randomly drawn samples of soldiers and sailors in a certain country'

	No. in the sample	Mean	SD
Soldiers	332	12.78	2.43
Sailors	615	12.99	2.48

Is the difference between the mean scores significant?

23. The records of a certain hospital showed the birth of 723 males and 617 females in a certain week . Do these conform to the hypothesis that the sexes are born in equal proportions $(\alpha = 0.05)$

24. In two colleges affiliated to a university 46 out of 200 and 48 out of 250 candidates failed in an examination. If the percentage of failure in the university is 18% examine whether the colleges differ significantly.

25. Survey of 320 families with 5 children each revealed the following distribution

No. of boys	5	4	3	2	1	0
No. of girls	0	1	2	3	4	5
No. of families	14	56	110	88	40	12

Is the result consistent with the hypothesis that the male & female births are equally probable.

26. Develop the large sample test for testing the equality of proportions.

27. Explain how the χ^2 distribution may be used to test the goodness of fit.

28. Describe how the χ^2 distribution may be used for testing homogeneity.

29. A sample of 400 observations were taken from a population with SD 15. If the mean of the sample is 27. Test whether the hypothesis that the mean of the population is less than 24($\alpha = 0.05$).

30. The mean weight of a sample of students was 48 kgs with SD 5 kgs. What is the minimum size of the sample if the hypothesis that the mean weight of students is 510 was rejected at 5% level of significance?

31. 12 rats were given a high protein diet & another set of 7 rats given a loss protein diet. The gain in weight in gms observed in two sets are given below.

High protein Diet: 13 14 10 11 12 16 10 8 11 12 9 12

Low protein Diet: 7 11 8 10 10 13 9

Examine whether the high protein diet is superior to the low pRotein diet at 5% level of significance.

32. Explain the small sample test and large sample tests.Distinguish between their application roles with illustration.

33. Describe the method of testing a simple hypothesis. $H_0: \mu = \mu_0$ against the alternative $H_1: \mu \neq \mu_0$ in a normal population with μ and variance unity.

34. A sample of size 8 from a normal population with SD 3 is 6,8,11,5,9,11,10,12 examine whether the mean of the population is 7.

35. Explain linear model.

36. Explain a contingency table using examples.

37. Explain the difference between the laboratory experiments & field experiments illustrated with suitable examples.
38. Explain the algebraic method of the partitioning of the total sum of squares in two way classification.
39. What is ANOVA? Explain how the total sum of squares is analysed & tested fprr one way classification.Write down the ANOVA table.
40. Distinguish between assignable causes of variation and random causes of variation.Give examples in which (a) only assignable causes are responsible for variation (b) only random causes are responsible for variation

15 Marks Questions

1. Explain the desirable properties of a good point estimator with examples.
2. If 8.6,7.9,8.3,6.4,8.4,9.8,7.2,7.8,7.5 are the observed values of random values of a random samples of size 9 from N(μ, σ^2). Derive & estimate 95% confidence interval for μ&σ^2.
3. The following random samples are measurements of heat producing capacity of coal in two mines construct (a)90% (b)95% (c)99% confidence interval for the difference of means

Mine 1: 7250,7025,7245,7050,7235

Mine 2: 8960,8890,8000,8250,8687,8840

4. A random sample of 20bullets produced by a machine shows an average diameter of 3.5 nm & SD of 0.2mm. Assuming that the diameter measurements follows N(μ, σ^2) obtain a 95% interval estimate for the mean and 99% interval estimate for the true variance.

5. Write the principle steps involved in statistica; tests.Explain the procedure for testing the equality of means and equality of proportions of a large population.
6. Define (a) acceptance error & (b) rejection error in testing a hypothesis. Compute these errors when the hypothesis p=⅓ is tested against the alternative hypothesis p=⅔ in 10 binomial trials if the critical region consists of points x= 8,9,10where x is the number of success and p the probability of success in a single trial.
7. Give the frequency function f(x,θ)=1/θ, 0≤ x ≤ θand 0 elsewhere if you are testing the hypothesis$H_0: \theta = 1$against $H_1: \theta = 2$ by means of a single observation x, what would be size of type 1 error & type 2 error if we choose the interval (a)0.5≤ x (b)1≤ x≤1.5 as the critical regions?
8. In a city the milk consumption of families x, is assumed to follow the distribution, f(x,θ)=$1/\theta\ e^{\frac{-x}{\theta}}$ 0≤x≤∞,θ>0. The hypothesis $H_0: \theta = 5$ is rejected in favour of H_1:θ=10 if a family is selected at random consumes 15 units or more, Obtain the critical region and sizes of two types of errors.
9. X is normally distributed with $\sigma = 10$ and we want to test $H_0: \mu = 100$ against $H_1:\mu = 110$. The critical region suggested is $\bar{x} \geq k$ where $\bar{x}$ is the mean of the sample chosen. How large a sample should be taken if the significance level is 0.05 and the probability of second type of error is 0.02?
10. In a consignment containing 10 articles θ are defective. The hypothesis $H_0: \theta = 5$ is rejected in favour of $H_1: \theta = 1$ if (a) two articles selected at random with replacement are both

defective or both nondefective (b) two articles selected at random with replacement are one defective & one non defective. Determine the significance level and power of both the tests.

11. Before an increase in excise duty on tea 400 people out of a sample of 500 persons were found to be tea drinkers. After an increase in duty 400 persons were found to be tea drinkers in a sample of 600 people. Examine whether there is any significant decrease in consumption of tea because of the increase in excise duty.

12. In a Sample of 400 students from the university college 75%were found to have passed in an examination but in a sample of 500 students from the affiliated colleges, only 60% were found to have secured the pass minimum.Does this conclusively establish the superiority of the university college.

13. It is claimed that children inherit their eye colours from fathers more than mothers. On the basis of the following data do you agree with this observation?

	Child's eye colour	
	Blue	Not blue
Father's blue	20	15
Mother's blue	10	7

14. The manufactures of an automatic sugar bagging machine claims that the variance of the bag weights is less than 0.01. Do the following observation of the weights of a randomly

chosen sample of bags support the claim ($\alpha = 0.05$)
10.1,9.8,10.1,9.8,10.0,9.7,9.9,10.0,9.8.

15. It is known that the mean diameters of rivets produced by two firms A & B are pacticall the same but the standard deviations may differ. For 22 rivets produced by firm A, the SD is 2.9 mm while for 16 rivets manufactured by firm B , the SD is 3.8 mm. Calculate the statistic you could use to test whether the products of firm A have the same variability as those of firm B.

16. 5Tests were carried out to assess the strength of single fibre yarn spun on two different machines A&B. The results are presented below.

Machine A : 4.0 4.4 3.9 4.0 4.2 4.4 5.0 4.8 4.6 3.9

Machine B :5.3 4.3 4.1 4.4 5.3 4.2 3.8 3.9 5.4 4.6

(1)Obtain unbiased estimates for the variances σ_A^2 & σ_B^2

(2)Assume that the samples have been taken from normal populations and test the hypothesis that the variability is the same for both the machines.

17. A test was given to 5 students taken at random from the fifth standard of three schools of a town. The scores are given below. Carry out the analysis of variance and state your conclusion.

School 1: 9 7 6 5 8

School 2: 7 4 5 4 5

School 3: 6 5 6 7 6

18. The following table gives the yields of wheat in 30 test plots which are given 3 different fertilizers. Test whether the fertilizers are equal in their effects.

Fertilizer 1: 50 60 60 65 70 80 75 80 85 75

Fertilizer 2: 60 60 65 70 75 80 70 75 85 80

Fertilizer 3: 40 50 50 60 60 60 65 75 70 70

19. Samples from 3 varieties of coal were analysed for the ash content and the results obtained are given below.

    A: 8 5 5

    B: 7 6 4 4

    C: 3 6

Do the varieties differ significantly in ash content.

20. A medical experiment was made to test the additional hours of sleep due to 3 drugs A,B,C tried on patients from four different age groups. Examine whether the three drugs as well as the four age groups are similar in their effect on gain of sleep.

	Age group			
	30-40	40-50	50-60	60-70
Drug A	2.0	1.2	1.0	0.3
Drug B	1.1	0.8	0.0	-0.1
Drug C	1.5	1.3	3.9	0.1