VTM NSS COLLEGE DHANUVACHAPURAM

Department of Mathematics

Question Bank

4th Semester Maths for Physics

MM 1431.1 Fourier series, Complex Analysis and Probability Theory.

2 marks questions

1. Write down the Euler formula for calculating the fourier coefficient.

2. Find a Fourier series to represent x^2 in the interval $(-l, l)$

3. Express $f(x) = x$ as a half-range cosine series in $0 < x < 2$.

4. Find the fourier transform of $e^{-|x|}$.

5. Write the Dirichlet's Conditions in a fourier series

6. Find the fourier transforms of $f(x) = \begin{cases} 1 & \text{for } |x| < 1 \\ 0 & \text{for } |x| > 1 \end{cases}$

7. State and prove modulation theorem of fourier transforms.

8. Show that $F_s[x f(x)] = -\frac{d}{ds}\{F_c(s)\}$ and

$$F_c[x f(x)] = \frac{d}{ds}\{F_s(s)\}$$

9. Find the fourier sine and cosine transform of $x e^{-ax}$.

10. Find the fourier sine transform of $\frac{e^{-ax}}{x}$.

11. In 256 sets of 12 tosses of a coin, In how many cases one can expect 8 heads and 4 tails.

12. How many 4 digit numbers can be formed from the 6 digits 2, 3, 5, 6, 7, 9 without repetition. How many of ~~13.~~ them are less than 500 ?

13. A committee consists of 9 students 2 of which are from first year, 3 from second year and 4 from third year. 3 students are to be removed at random. What is the chance that

(i) the 3 students belong to different classes.

(ii) two belong to the same class and third to a different class.

14. let A and B be two events with $P(A) = 12$, $P(B) = 13$ $P(AB) = 14$. find $P(A'/B')$.

15. What is the chance of that a leap year selected at random will contain 53 sundays ?

16. Find the probability of getting a king of red colour from a well shuffled deck of 52 cards?

17. Evaluate $P(A/B)$ and $P(B/A)$ given $P(A) = 1/4$ and $P(B) = 1/3$

18. use a binomial distribution to calculate $P(X=0)$ and $P(X=1)$

19. Suppose 5 cards are drawn at random from a pack of 52 cards. If all cards red, find the probability that all ~~of~~ them are hearts.

20. For any two events A and B prove that $P(A \cap B') = P(A) - P(A \cap B)$

21. Find the probability of drawing an ace or a spade or both from a deck of cards.

22. Write any three axioms of probability.

23. Define an entire function. Give an example.

24. show that $f(x, y) = x^2 - y^2$ is a harmonic function.

25. Evaluate $\int_C \frac{dz}{z-1}$, where C is the circle $|z-1| = 2$.

26. Show that $\frac{e^{1/z}}{z^2}$ has an essential singularity at $z=0$.

27. Evaluate $\int_C \frac{z^2}{z-3}$ where C is the circle $|z|=1$

28. Find the real values of x, y so that $-3+ix^2y$ and x^2+y+4i may represent complex Conjugate numbers.

29. Simplify

$$\frac{(\cos 3\theta + i\sin 3\theta)^4 (\cos 4\theta - i\sin 4\theta)^5}{(\cos 4\theta + i\sin 4\theta)^3 (\cos 5\theta + i\sin 5\theta)^{-4}}$$

30. show that $\int_C \frac{dz}{z-a} = 2\pi i$

31. Determine the region in the z-plane represented by

(i) $1 < |z+2i| \leq 3$

(ii) $R(z) > 3$

32. If z_1, z_2 be two complex numbers, show that

$$(z_1+z_2)^2 + (z_1-z_2)^2 = 2\left[|z_1|^2 + |z_2|^2 \right]$$

33. If $w = \log z$. Find $\frac{dw}{dz}$ and determine where w is non-analytic.

34. Evaluate $\oint_C \frac{\sin^2 z}{(z-\pi/6)^3} dz$, where C is the circle $|z|=1$

~~35. Expand sinz in a Tay~~

35. What type of singularity have the following function $\frac{1}{1-e^z}$

4 marks questions

36. Express $f(x)=x$ as a half range sine series in $(0,2)$

37. obtain the Fourier series for $f(x)=e^{-x}$ in the interval $0<x<2\pi$.

38. Find the Fourier series for the function

$$f(t)=\begin{cases} -1 & \text{for } -\pi<t<-\pi/2 \\ 0 & \text{for } -\pi/2<t<\pi/2 \\ 1 & \text{for } \pi/2<t<\pi \end{cases}$$

39. If $f(x)=|\cos x|$. Expand $f(x)$ as a fourier series in the interval $(-\pi,\pi)$

40. Find the Fourier cosine transform of $f(x)=\dfrac{1}{1+x^2}$

41. Two cards are drawn in succession from a pack of 52 cards. Find the chance that the first is a king and second queen if the first card is

(a) replaced

(b) not replaced

42. Three identical boxes contain red and white balls. The first box contains 3 red and 2 white balls, the second box has 4 red and 5 white balls, and the third box has 2 red and 4 white balls. A box

is chosen very randomly and a ball is drawn from it. If the ball that is drawn out of is red, what is the probability that the second box is chosen?

43. A die is tossed twice. A success is "getting 1 or 6" on a toss. Find the mean and variance of the number of successes.

44. A problem in mathematics is given to 3 students A, B, C whose chances of solving it are $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$ respectively. What is the probability that the problem will be solved.

45. (a) A biased coin is tossed till a head appears for the first time. What is the probability that the number of required tosses is odd.

(b) Two persons A and B toss an unbiased coin alternately on the understanding that the first who gets the head wins. If A starts the game. Find their respective chances of winning.

46. If A, B, C are any three events, then show that

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)$$

47. A bag contains 8 white and 6 red balls. Find the probability of drawing two balls of the same colour.

48. Write any four applications of Binomial distributions.

49. Show that $w = \log z$ is analytic everywhere except at $z=0$

50. Evaluate the $\int_C \frac{z^2-z+1}{z-1}\,dz$ where C is the circle

(i) $|z|=1$ (ii) $|z| = \frac{1}{2}$

51. Expand $\sin z$ in a taylor's series about $z=0$

52. Find the sum of residues of $f(z) = \frac{\sin z}{z\cos z}$ at its poles inside the circle $|z|=2$.

53. If the probability of a bad reaction from a certain injection is 0.001. Determine the chance that out of 2,000 individuals more than two will get a bad reaction.

54. Two cards are drawn in succession from a pack of 52 cards. Find the chance that the first is a king and the second a queen if the first card is (i) replaced (ii) not replaced.

55. A and B throw alternately with a pair of dice. A wins if he throws 6 before B throws 7 and B wins if he throws 7 before A throws 6. If A begins, find his chance of winning.

56. Evaluate $\oint_c \frac{e^z}{(z^2+\pi^2)^2} dz$ where c is $|z|=4$

57. Evaluate $\int_0^{2+i} (\bar{z})^2 dz$, along (i) the line $y = x/2$ (ii) the real axis to 2 and then vertically to $2+i$.

58. Obtain the Fourier expansion of $x \sin x$ as a cosine series in $(0, \pi)$

59. Reduce $1-\cos\alpha + i\sin\alpha$ to the modulus amplitude form.

60. Find the equation whose roots are $2\cos \pi/7$, $2\cos 3\pi/7$, $2\cos 5\pi/7$.

61. State and Prove linearity of Fourier Transforms

62. Find the Fourier Cosine transform of

$$f(x) = \begin{cases} x & \text{for } 0<x<1 \\ 2-x & \text{for } 1<x<2 \\ 0 & \text{for } x>2 \end{cases}$$

15 marks questions

63. (i) obtain the fourier series for $f(x) = e^{-x}$ in the interval $0 < x < 2\pi$.

(ii) Find the Fourier transform of $e^{-a^2x^2}$, $a > 0$

64. Find the fourier sine transform of $e^{-|x|}$.

Hence show that $\displaystyle\int_0^\infty \frac{x \sin mx}{1+x^2}\,dx = \frac{\pi e^{-m}}{2}$, $m > 0$

65. Find the Fourier transform of :

$$f(x) = \begin{cases} 1 - x^2, & |x| \le 1 \\ 0, & |x| > 1 \end{cases}$$

Hence evaluate $\displaystyle\int_0^\infty \frac{x\cos x - \sin x}{x^3} \cos\frac{x}{2}\,dx$.

66. Find the fourier series expansion for $f(x)$, if

$$f(x) = \begin{cases} -\pi, & -\pi < x < 0 \\ x, & 0 < x < \pi \end{cases}$$

Deduce that $\dfrac{1}{1^2} + \dfrac{1}{3^2} + \dfrac{1}{5^2} + \cdots = \dfrac{\pi^2}{8}$

67. Find the Fourier series for

$$f(t) = \begin{cases} 0 & -2 < t < -1 \\ 1+t & -1 < t < 0 \\ 1-t & 0 < t < 1 \\ 0 & 1 < t < 2 \end{cases}$$

68. (i) There are three bags : first containing 1 white, 2 red, 3 green balls, second 2 white, 3 red, 1 green balls and third 3 white, 1 red, 2 green balls. Two balls are drawn from a bag chosen at random. These are found to be 1 white and 1 red. Find the probability that the balls so drawn came from the second bag.

(ii) In a sampling a large number of parts manufactured by a machine, the mean number of defectives in a sample of 20 is 2. Out of 1000 such samples, how many would be expected to contain atleast 3 defective parts.

69. A biased coin is tossed till a head appears for the first time.

(a) What is the probability that the number of required tosses is odd.

(b) Two persons A and B toss an unbiased coin alternatively understanding that the first who gets the head wins. If A starts then find their respective chance of winning.

70. A random variable X has the following probability function.

x :	0	1	2	3	4	5	6	7
$P(x)$:	0	k	$2k$	$2k$	$4k$ $3k$	k^2	$2k^2$	$7k^2+k$

(a) Find the value of k

(b) Evaluate

(i) $P(X<6)$

(ii) $P(X \geq 6)$ and

(iii) $P(0<X<5)$

71. (i) The odds that a book will be reviewed favourably by three independent critics are 5 to 2, 4 to 3 and 3 to 4. What is the probability that of the three reviews, a majority will be favourable.

(ii) Given $P(A) = \frac{1}{4}$ $P(B) = \frac{1}{3}$ and $P(A \cup B) = \frac{1}{2}$. Evaluate $P(A/B)$, $P(B/A)$, $P(A \cap B')$ and $P(A/B')$.

72. The following data are the number of seeds germinating out of 10 on damp filter paper for 80 sets of seeds. Fit a binomial distribution to these data.

x :	0	1	2	3	4	5	6	7	8	9	10
f :	6	20	28	12	8	6	0	0	0	0	0

73. (i) If $f(z)$ is an analytic function with constant modulus, then show that $f(z)$ is a constant.

(ii) Evaluate, using Cauchy's Integral $\int_C \frac{e^z}{(z^2+\pi^2)^2} dz$ where C is the circle $|z|=4$.

74. By integrating around a unit circle, evaluate

$$\int_0^{2\pi} \frac{\cos 3\theta}{5-4\cos\theta} d\theta$$

75. Find the residue of $f(z) = \frac{z^3}{(z-1)^4(z-2)(z-3)}$ at its poles and hence evaluate $\oint_C f(z)dz$ where C is the circle $|z| = 2.5$

76. Evaluate $\int_0^{\infty} \frac{\sin mx}{x} dx$ when $m>0$.

77. Prove that $\int_0^{\infty} \sin x^2 dx = \int_0^{\infty} \cos x^2 dx = \frac{1}{2}\sqrt{\frac{\pi}{2}}$

78. Evaluate $\int_{-\infty}^{\infty} \frac{x^2 dx}{(x^2+1)(x^2+4)}$

79. Expand $f(z) = \frac{1}{(z-1)(z-2)}$ in the region

(a) $|z|<1$

(b) $1<|z|<2$

(c) $|z|>2$

80. Expand $f(x) = \sqrt{(1-\cos x)}$, $0<x<2\pi$ in a Fourier series. Hence evaluate

$$\frac{1}{1.3}+\frac{1}{3.5}+\frac{1}{5.7}+\cdots$$

81. If $f(x) = \begin{cases} 0 & -\pi \le x \le 0 \\ \sin x & 0 \le x \le \pi \end{cases}$. Prove that

$$f(x) = \frac{1}{\pi}+\frac{\sin x}{2}-\frac{2}{\pi}\sum_{n=1}^{\infty}\frac{\cos 2nx}{4n^2-1}$$

Hence show that $\frac{1}{1.3}-\frac{1}{3.5}+\frac{1}{5.7}-\cdots-\infty = \frac{1}{4}(\pi-2)$

82. Obtain Fourier series for the function $f(x)$ given by $f(x) = \begin{cases} 1+\frac{2x}{\pi} & -\pi \le x \le 0 \\ 1-\frac{2x}{\pi} & 0 \le x \le \pi \end{cases}$

Deduce that $\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\cdots = \frac{\pi^2}{8}$

83. Find the Fourier sine and cosine transform of x^{n-1}, $n>0$ and prove that $\frac{1}{\sqrt{x}}$ is self-reciprocal under Fourier sine and cosine transforms.

84. (a) Find the Fourier transform of $e^{-a^2x^2}$, $a<0$. Hence deduce that $e^{-x^2/2}$ is self reciprocal in respect of Fourier transform.

(b) Find Fourier transform of

(i) $e^{-2(x-3)^2}$

(ii) $e^{-x^2} \cos 3x$.

85. (a) Find the locus of the point z, when

(i) $\left|\frac{z-a}{z-b}\right| = k$

(ii) $\operatorname{amp}\left(\frac{z-a}{z-b}\right) = \alpha$ where k and α are constants.

~~8~~ (b) Show that the equation of the ellipse having foci at z_1, z_2 and major axis $2a$, is

$$\cancel{|z_1 - z_2} \quad |z-z_1| + |z-z_2| = 2a.$$

86. If $\sin\alpha + \sin\beta + \sin\gamma = \cos\alpha + \cos\beta + \cos\gamma = 0$

Prove that

(i) $\sin 2\alpha + \sin 2\beta + \sin 2\gamma = 0$

(ii) $\sin 3\alpha + \sin 3\beta + \sin 3\gamma = 3\sin(\alpha+\beta+\gamma)$

(iii) $\sin 4\alpha + \sin 4\beta + \sin 4\gamma = 2\sum \sin 2(\alpha+\beta)$

(iv) $\sin(\alpha+\beta) + \sin(\cancel{\alpha}\,\beta+\gamma) = \sin(\gamma+\alpha) = 0$

87. (i) Show that the roots of the equation $(x-1)^n = x^n$, n being a positive integer are $\frac{1}{2}\left(1+i\cot\frac{r\pi}{n}\right)$ where ~~n~~ r has the values $1, 2, 3, \ldots n-1$

(ii) Find the 7th roots of unity and prove that the sum of their nth powers always vanishes unless n be a multiple number of 7, n being an integer, and then the sum is 7.

88. Evaluate $\int_C (z^2+3z+2)\,dz$ where C is the arc of the cycloid $x = a(\theta+\sin\theta)$, $y = a(1-\cos\theta)$ between the points $(0,0)$ and $(\pi a, 2a)$

89. Evaluate, using Cauchy's integral formula

$$\oint_C \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)(z-2)}\,dz$$ where C is the circle $|z| = 3$

90. Verify Cauchy's theorem by integrating e^{iz} along the boundary of the triangle with the vertices at the points $1+i$, $-1+i$, and $-1-i$.

91. If $F(\xi) = \oint_C \frac{4z^2+z+5}{z-\xi}\,dz$, where C is the ellipse $\left(\frac{x}{2}\right)^2 + \left(\frac{y}{3}\right)^2 = 1$. Find the value of

(a) $F(3.5)$ (b) $F''(-1)$ c) $F''(-i)$

92. (i) State Residue theorem

(ii) Determine the poles of the function

$$f(z) = \frac{z^2}{(z-1)^2(z+2)}$$ and the residue at each pole.

93. Evaluate $\int_{-\infty}^{\infty} \frac{e^{ax}}{e^x+1}\,dx$

94. Evaluate $\int_0^{\infty} \frac{\cos ax}{x^2+1}\,dx$.

95. Show that $\int_0^{\infty} \frac{x^{p-1}}{1+x}\,dx = \frac{\pi}{\sin p\pi}$, $0<p<1$

96. Three machines M_1, M_2, M_3 produce identical items. Of their respective output 5%, 4% and 3% of items are faulty. On a certain day, M_1 has produced 25% of the total output, M_2 has produced 30% and M_3 the remainder. An item

selected at random is found to be faulty. What are the chances that it was produced by the machine with the highest output?

~~97. the following data are the number of~~ seeds

~~geo~~

97. Fit a Poisson distribution to the set of observations

x :	0	1	2	3	4
f :	122	60	15	2	1

98. A can hit a target 3 times in 5 shots, B 2 times in 5 shots and C 3 times in 4 shots. They fire a volley. What is the probability that
(i) two shots hit
(ii) atleast two shots hit

99. The students in a class are selected at random, one after the other, for an examination. Find the probability P that the boys and girls in the class alternate if
(i) the class consists of 4 boys and 3 girls

(ii) the class consists of 3 boys and 3 girls.

100. (i) Given $P(A) = 1/2$ $P(B) = 1/3$ $P(AB) = 1/4$

find the value of $P(A+B)$

(ii) the probability density function of a variate X is

X :	0	1	2	3	4	5	6
P(X) :	k	3k	5k	7k	9k	11k	13k

Find $P(X<4)$, $P(X \geq 5)$, $P(X \leq 6)$ and find the minimum value of k so that $P(X \leq 2) > 0.3$