(Pages : 6) **N – 7786**

Reg. No. : ....

Name : ....

Fourth Semester B.Sc. Degree Examination, August 2022

First Degree Programme under CBCSS

Mathematics

Complementary Course for Chemistry/Polymer Chemistry

MM 1431.2 : MATHEMATICS IV — DIFFERENTIAL EQUATIONS, VECTOR CALCULUS AND ABSTRACT ALGEBRA

(2019 Admission Onwards)

Time : 3 Hours Max. Marks : 80

PART – A

All the first **10** question are compulsory. Each carry **1** mark:

1. A generalisation of the homogeneous ODE is known as ————

2. What is the form of Bernoulli's equation?

3. Write the Clairaut's equation

4. Write the general linear recurrence relation formula.

5. Say true or false: A line integral is also known as a path integral.

6. The vector area of a surface S is ————

7. Let V is a small volume enclosing P and S is its bounding surface. If ϕ is a scalar field and a is a vector field then at any point P, $\nabla \times a =$ ————.

VTM NSS COLLEGE LIBRARY

8. Write the cancellation law in a group.

9. The order of the identity of a group is ________

10. Say true or false: if groups G and G' are isomorphic, G and G'' are isomorphic, then G' and G'' are isomorphic.

(10 × 1 = 10 Marks)

PART – B

Answer **any eight** questions. These questions carry **2** marks each.

11. Solve $\frac{dy}{dx} + 2xy = 4x$.

12. Solve $\frac{dy}{dx} + x + xy$.

13. Find a particular integral of the equation $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = e^x$.

14. State Green's theorem in a plane.

15. Find a solution of $(x^2 + x)\frac{dy}{dx}\frac{d^2y}{dx^2} - x^2y\frac{dy}{dx} - x\left(\frac{dy}{dx}\right)^2 = 0$.

16. Solve $\frac{dy}{dx} = (x + y + 1)^2$.

17. Find the volume enclosed between a sphere of radius a centred on the origin and a circular cone of half-angle α with its vertex at the origin.

VTM NSS COLLEGE LIBRARY

18. Reduce $\int_C \mathbf{a} \cdot d\mathbf{r}$ to a set of scalar integrals by writing the vector field **a** in terms of its Cartesian components as $\mathbf{a} = a_x\mathbf{i} + a_y\mathbf{j} + a_z\mathbf{k}$, where a_x, a_y, a_z are each (in general) functions of x, y, z.

19. Write the line integral for the total work done by a force F when it moves its point of application from a point A to a point B along a given curve C.

20. When do you say that a plane region R is simply connected?

21. Find an expression for the angular momentum of a solid body rotating with angular velocity ω about an axis through the origin.

22. Show that the inverse of any particular element of a group $\mathcal{G}$ is unique.

23. Define cyclic group.

24. Write three properties of the subgroups of a group $\mathcal{G}$.

25. Define an equivalence relation on a set S.

26. Let $\mathcal{G}$ be a group and $\mathcal{H}$ be a subgroup of $\mathcal{G}$. Prove that two cosets of $\mathcal{H}$ are either disjoint or identical.

(8 × 2 = 16 Marks)

PART – C

Answer **any six** questions. These questions carry **4** marks each.

27. Solve : $\frac{dy}{dx} = \frac{-2}{y} - \frac{3y}{2x}$.

28. Solve $\frac{dy}{dx} + \frac{y}{x} = 2x^3y^4$.

VTM NSS COLLEGE LIBRARY

29. Solve $6y^2p^2 + 3xp - y = 0$.

30. Solve $4x^2 \frac{d^2y}{dx^2} + 4x\frac{dy}{dx} + (x^2 - 1)y = 0$.

31. Evaluate the line integral $I = \int_C (x - y)^2\, ds$, where C is the semicircle of radius a running from $A = (a, 0)$ to $B = (-a, 0)$ and for which $y \geq 0$.

32. Find the vector area of the surface of the hemisphere $x^2 + y^2 + z^2 = a^2$ with $z \geq 0$.

33. Use the divergence theorem to prove $\int_V \nabla \times b\, dV = \oint_S dS \times b$.

34. From Ampere's law, derive Maxwell's equation in the case where the currents are steady, i.e. $\nabla \times B - \mu_0 J = 0$.

35. Let $\Phi : \mathcal{G} \rightarrow \mathcal{G}'$ be a homomorphism of $\mathcal{G}$ into $\mathcal{G}'$ then show that the set of elements $\mathcal{H}'$ in $\mathcal{G}'$ that are images of the elements of $\mathcal{G}$ forms a subgroup of $\mathcal{G}'$.

36. Show that the traces of equivalent matrices are equal.

37. Show that the field of all non-zero complex numbers that have unit modulus are of the form $e^{i\theta}$ where $0 \leq \theta < 2\pi$, form a group under multiplication.

38. For the hydrogen molecule consists of two atoms H of hydrogen, what are different sets of operations rotations, reflections, and inversions.

(6 × 4 = 24 Marks)

VTM NSS COLLEGE LIBRARY

PART – D

Answer **any two** questions. These questions carry **15** marks each.

39. (a) Find the value of u_{16} if the series u_n satisfies $u_{n+1} + 4u_n + 3u_{n-1} = n$ for $n \geq 1$, with $u_0 = 1$ and $u_1 = -1$.

(b) Solve $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 2e^{-x}$ subject to the boundary conditions $y(0) = 2$, $y'(0) = 1$.

40. (a) Solve $\frac{d^2y}{dx^2} + y = \operatorname{cosec} x$.

(b) Use Green's functions to solve $\frac{d^2y}{dx^2} + y = f(x)$, subject to the one-point boundary conditions $y(0) = y'(0) = 0$.

41. Evaluate the line integral $I = \int_C \mathbf{a} \cdot d\,\mathbf{r}$, where $a = (x+y)\mathbf{i} + (y-x)\mathbf{j}$, along

(a) the parabola $y^2 = x$ from (1, 1) to (4, 2),

(b) the curve $x = 2u^2 + u + 1$, $y = 1 + u^2$ from (1,1) to (4,2),

(c) the line y = 1 from (1, 1) to (4, 1), followed by the line x = 4 from (4, 1) to (4, 2).

VTM NSS COLLEGE LIBRARY

42. (a) Show that the area of a region R enclosed by a simple closed curve C is given by

$$A = \frac{1}{2}\oint_C (xdy - ydx) = \oint_C xdy = -\oint_C ydx$$. Hence calculate the area of the ellipse $x = a\cos\varphi,\ y = b\sin\varphi$.

(b) Show that the geometrical definition of grad leads to the usual expression for ∇_{φ} in Cartesian coordinates.

43. Determine the irreps contained in the representation of the group 3 m in the vector space spanned by the functions x^2, y^2, xy.

44. If n_μ is the dimension of the μth irrep of a group $\mathcal{G}$ then show that $\sum_{\mu} n_{\mu}^2 = g$, where g is the order of the group.

(2 × 15 = 30 Marks)

VTM NSS COLLEGE LIBRARY