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Reg. No. : ....

Name : ....

Fourth Semester B.Sc. Degree Examination, August 2022

First Degree Programme under CBCSS

Mathematics

Complementary Course for Chemistry / Polymer Chemistry

MM 1431.2 : MATHEMATICS IV — DIFFERENTIAL EQUATIONS, VECTOR CALCULUS AND ABSTRACT ALGEBRA

(2018 Admission)

Time : 3 Hours Max. Marks : 80

PART – A

All the first **10** question are compulsory. Each carry **1** mark.

1. A generalisation of the homogeneous ODE is known as ————

2. What is the form of Bernoulli's equation?

3. Write the Clairaut's equation

4. Write the general linear recurrence relation formula.

5. Say true or false: A line integral is also known as a path integral.

6. The vector area of a surface S is ————

7. Let V is a small volume enclosing P and S is its bounding surface. If ϕ is a scalar field and a is a vector field then at any point P, $\nabla \times a =$ ————.

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8. Write the cancellation law in a group.

9. The order of the identity of a group is ________

10. Say true or false: if groups $\mathcal{G}$ and $\mathcal{G}'$ are isomorphic, $\mathcal{G}$ and $\mathcal{G}''$ are isomorphic, then $\mathcal{G}'$ and $\mathcal{G}''$ are isomorphic.

(10 × 1 = 10 Marks)

PART – B

Answer **any eight** questions. These questions carry **2** marks each.

11. Solve $\frac{dy}{dx} + 2xy = 4x$.

12. Find a particular integral of the equation $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = e^x$.

13. Find a solution of $(x^2 + x)\frac{dy}{dx}\frac{d^2y}{dx^2} - x^2 y\frac{dy}{dx} - x\left(\frac{dy}{dx}\right)^2 = 0$.

14. Solve $\frac{dy}{dx} = (x + y = 1)^2$.

15. Find the volume enclosed between a sphere of radius a centred on the origin and a circular cone of half-angle α with its vertex at the origin.

16. Write the line integral for the total work done by a force F when it moves its point of application from a point A to a point B along a given curve C.

17. When do you say that a plane region R is simply connected?

18. Find an expression for the angular momentum of a solid body rotating with angular velocity ω about an axis through the origin.

19. Show that the inverse of any particular element of a group $\mathcal{G}$ is unique.

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20. Write three properties of the subgroups of a group $\mathcal{G}$.

21. Define an equivalence relation on a set S.

22. Let $\mathcal{G}$ be a group and $\mathcal{H}$ be a subgroup of $\mathcal{G}$. Prove that two cosets of $\mathcal{H}$ are either disjoint or identical.

(8 × 2 = 16 Marks)

PART – C

Answer **any six** questions. These questions carry **4** marks each.

23. Solve $\frac{dy}{dx} + \frac{y}{x} = 2x^3y^4$.

24. Solve $6y^2p^2 + 3xp - y = 0$.

25. Solve $4x^2\frac{d^2y}{dx^2} + 4x\frac{dy}{dx} + (x^2 - 1)y = 0$.

26. Evaluate the line integral $I = \int_c (x - y)^2\, ds$, where C is the semicir'cle of radius a running from $A = (a, 0)$ to $B = (-a, 0)$ and for which $y \geq 0$.

27. Find the vector area of the surface of the hemisphere $x^2 + y^2 + z^2 = a^2$ with $z \geq 0$.

28. From Ampere's law, derive Maxwell's equation in the case where the currents are steady, i.e. $\nabla \times B - \mu_0 J = 0$.

29. Let $\Phi : \mathcal{G} \rightarrow \mathcal{G}'$ be a homomorphism of $\mathcal{G}$ into $\mathcal{G}'$ then show that the set of elements $\mathcal{H}'$ in $\mathcal{G}$ that are images of the elements of $\mathcal{G}$ forms a subgroup of $\mathcal{G}'$.

30. Show that the traces of equivalent matrices are equal

31. For the hydrogen molecule consists of two atoms H of hydrogen, what are different sets of operations rotations, reflections, and inversions.

(6 × 4 = 24 Marks)

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PART – D

Answer **any two** questions. These questions carry **15** marks each.

32\. (a) Find the value of u_{16} if the series u_n satisfies $u_{n+1} + 4u_n + 3u_{n-1} = n$ for $n \geq 1$, with $u_0 = 1$ and $u_1 = -1$.

(b) Solve $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 2e^{-x}$ subject to the boundary conditions $y(0) = 2$, $y'(0) = 1$.

33\. (a) Solve $\frac{d^2y}{dx^2} + y = \operatorname{cosec} x$.

(b) Use Green's functions to solve $\frac{d^2y}{dx^2} + y = f(x)$, subject to the one-point boundary conditions $y(0) = y'(0) = 0$.

34\. (a) Show that the area of a region R enclosed by a simple closed curve C is given by $A = \frac{1}{2}\oint_C (xdy - ydx) = \oint_C xdy = -\oint_C ydx$. Hence calculate the area of the ellipse $x = a\cos\varphi$, $y = b\sin\varphi$.

(b) Show that the geometrical definition of grad leads to the usual expression for $\nabla\varphi$ in Cartesian coordinates.

35\. Determine the irreps contained in the representation of the group 3 m in the vector space spanned by the functions x^2, y^2, xy.

(2 × 15 = 30 Marks)

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