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Reg. No. : ....

Name : ....

Fourth Semester B.Sc. Degree Examination, August 2022

First Degree Programme under CBCSS

Mathematics

Complementary Course for Chemistry/Polymer Chemistry

MM 1431.2 : MATHEMATICS IV (ABSTRACT ALGEBRA AND LINEAR TRANSFORMATIONS)

(2014-2017 Admission)

Time : 3 Hours Max. Marks : 80

PART – I

Answer **all** questions. Each question carries **1** mark.

1. Write the identity element in the group $M_2(\mathbb{R})$ of all 2×2 matrices under matrix addition.

2. What is the inverse of 6 in the group $\mathbb{Z}_{10}$.

3. In $\mathbb{Z}_7$, which element is not a generator?

4. Find the remainder when -49 is divided by 8 according to the division algorithm.

5. Compute the product $(12)(5)$ in $\mathbb{Z}_{23}$.

6. Give a vector linearly dependent to $\begin{bmatrix} 3 \\ 4 \end{bmatrix}$.

P.T.O.

7. Give a linear transformation from $\mathbb{R}^2$ to $\mathbb{R}$.

8. Give a transformation from $\mathbb{R}$ to $\mathbb{R}$, which is not linear.

9. Give an example of a dilation on $\mathbb{R}^2$.

10. If T is a linear transformation, find $T(0)$.

(10 × 1 = 10 Marks)

PART – II

Answer **any eight** questions. Each question carries **2** marks.

11. Is M_3 ($\mathbb{R}$) of all 3×3 matrices under matrix multiplication a group? Why?

12. Find all generators of $\mathbb{Z}_6$.

13. Find the cyclic subgroup of $\mathbb{Z}_{12}$ generated by 9.

14. Define a permutation and give one example.

15. Find the subgroups of $\mathbb{Z}_5$.

16. What are the units in the ring of all integers $(\mathbb{Z}, +, .)$?

17. Show that the vectors $\begin{bmatrix}1\\0\end{bmatrix}, \begin{bmatrix}2\\1\end{bmatrix}$ are linearly independent.

18. Find a standard matrix A for the transformation $T(x) = 3x$, for $x \in \mathbb{R}^2$.

19. If a set $S = \{v_1, \ldots, v_p\}$ in $\mathbb{R}^n$ contains the zero vector, then show that the set S is linearly dependent.

20. Give an example of a one-to-one linear transformation from $\mathbb{R}^2$ to $\mathbb{R}^2$.

21. Write $\begin{bmatrix}3\\4\end{bmatrix}$ as a linear combination of $\begin{bmatrix}1\\2\end{bmatrix}$ and $\begin{bmatrix}2\\0\end{bmatrix}$.

22. Give the standard matrix for the transformation on $\mathbb{R}^2$ given by the reflection through the x_1-axis.

(8 × 2 = 16 Marks)

PART – III

Answer **any six** questions. Each question carries **4** marks.

23. Give 4 different binary operations on the set $\{a, b\}$.

24. Show that a group cannot have more than one identity element.

25. Show that the Klein 4 group V is not cyclic.

26. If R is a ring with identity 0, then show that 0a = a0 = 0 for all $a \in R$.

27. Write two proper subfields of the field of all real numbers $(\mathbb{R}, +, .)$.

28. Determine if the columns of $A = \begin{bmatrix} 0 & 1 & 4 \\ 1 & 2 & -1 \\ 5 & 8 & 0 \end{bmatrix}$ are linearly independent.

29. Determine if the vectors $\begin{bmatrix} 5 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 7 \\ 2 \\ -6 \end{bmatrix}, \begin{bmatrix} 9 \\ 4 \\ -8 \end{bmatrix}$ are linearly independent.

30. Let $A = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$, and $T : \mathbb{R}^2 \to \mathbb{R}^2$ by $T(x) = Ax$. Find the images under T of $u = \begin{bmatrix} 1 \\ -3 \end{bmatrix}$, $v = \begin{bmatrix} a \\ b \end{bmatrix}$, $5u$ and $u + v$.

31. Find the standard matrix for the rotation transformation on $\mathbb{R}^2$.

(6 × 4 = 24 Marks)

PART – IV

Answer **any two** questions. Each question carries **15** marks.

32. (a) Let * be defined on $\mathbb{Q}^+$ by $a * b = \frac{ab}{2}$. Show that $(\mathbb{Q}^+, *)$ is an abelian group.

    (b) Show that the subset S of $M_n(\mathbb{R})$ of all invertible $n \times n$ matrices under matrix multiplication a group.

33. (a) Describe S_3 and write the group table.

    (b) Write down the orders of each element of the group S_3.

34. (a) Let $T(x_1, x_2) = (3x_1 + x_2, 5x_1 + 7x_2, x_1 + 3x_2)$. Show that T is a one-to-one linear transformation. Does T maps $\mathbb{R}^2$ onto $\mathbb{R}^3$?

(b) Show that if a set contains more vectors than there are entries in each vector, then the set is linearly dependent.

35. Let $A = \begin{bmatrix} 1 & -3 \\ 3 & 5 \\ -i & 7 \end{bmatrix}$, $u = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$, $b = \begin{bmatrix} 3 \\ 2 \\ -5 \end{bmatrix}$, $c = \begin{bmatrix} 3 \\ 2 \\ 5 \end{bmatrix}$ and define $T : \mathbb{R}^2 \to \mathbb{R}^3$ bt $T(x) = Ax$.

(a) Find $T(u)$

(b) Find an x in $\mathbb{R}^2$ such that $T(x) = b$.

(c) Is there more than one x such that $T(x) = b$.

(d) Determine if c is in the range of T.

(2 × 15 = 30 Marks)