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Reg. No. : ....

Name : ....

First Semester B.Sc. Degree Examination, March 2023

First Degree Programme under CBCSS

Mathematics

Complementary Course I for Chemistry and Polymer Chemistry

MM 1131.2 : MATHEMATICS I - DIFFERENTIAL CALCULUS AND SEQUENCES AND SERIES

(2021 Admission onwards)

Time : 3 Hours Max. Marks : 80

SECTION – A

All the first **10** questions are compulsory. They carry **1** mark each.

1. Find $\lim_{x \to 2}(x^2 - x + 1)$.

2. State product rule for differentiation.

3. Evaluate $\log_2 5$ in terms of natural logarithms.

4. Find the domain of the function $f(x, y) = \dfrac{ln(x + y + 1)}{y - x}$.

5. Find $\dfrac{\partial f}{\partial x}$ and $\dfrac{\partial f}{\partial y}$ if $f(x, y) = x^2 y + 5y^3$.

6. Define an inflection point of a function.

7. Using L'Hôpital's rule, find $\lim_{x \to 0} \frac{\sin 2x}{x}$.

8. State the Extreme-Value Theorem.

9. Evaluate $\sum_{n=0}^{\infty} \frac{(-1)^n 5}{4^n}$.

10. Show that the sequence $\left\{(-1)^{n+1} \frac{1}{n}\right\}$ converges by finding the limit.

(10 × 1 = 10 Marks)

SECTION – B

Answer any **eight** questions. These question carries **2** marks each.

11. Show that the function f defined by $f(x) = \sqrt{4 - x^2}$ is continuous on the closed interval $(-2,2)$.

12. Find the derivative of $f(x) = \frac{2x^2 + x}{x^3 - 1}$

13. Find the derivative of $f(x) = ln\sqrt{x^2 + 1}$.

14. State Rolle's Theorem and verify it for the function $f(x) = x^3 - x$ for $x \in [-1,1]$.

15. Find all critical points of $f(x) = x^3 - 3x + 1$.

16. Evaluate $\lim_{x \to 0^+} x \, ln \, x$.

17. Define level surface for a function $f(x, y, z)$. Describe the level surfaces of $f(x, y, z) = x^2 + y^2 + z^2$.

18. Find the local linear approximation to $f(x, y) = \sqrt{x^2 + y^2}$ at $(3,4)$.

19. Express $\frac{\partial w}{\partial r}$ and $\frac{\partial w}{\partial s}$ in terms of r and s if $w = x + 2y + z^2$, $x = \frac{r}{s}$, $y = r^2 + \ln s, z = 2r$.

20. Find the Maclaurin series for e^x.

21. Find the sum of the series $\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$.

22. Use the alternating series test to check the convergence of $\sum_{k=1}^{\infty} (-1)^{k+1} \frac{1}{k}$..

(8 × 2 = 16 Marks)

SECTION – C

Answer any **six** questions. These questions carries **4** marks.

23. Evaluate $\lim_{x \to 1} \frac{x^2 + x - 2}{x^2 - x}$.

24. Evaluate $\frac{d}{dx} \sec^{-1}(5x^4)$.

25. Find the intervals on which the function $f(x) = x^{\frac{4}{3}} - 4x^{\frac{1}{3}}$ is increasing and decreasing.

26. Find $\lim_{x \to 0} (1 + \sin x)^{\frac{1}{x}}$. Let $y = (1 + \sin x)^{\frac{1}{x}}$.

27. Use chain rule to find $\frac{\partial w}{\partial u}$ and $\frac{\partial w}{\partial v}$, where $w = e^{xyz}$, $x = 3u + v$, $y = 3u - v$, $z = u^2 v$.

28. Find the slope of the sphere $x^2 + y^2 + z^2 = 1$ in the y-direction at the points $\left(\frac{2}{3}, \frac{1}{3}, \frac{2}{3}\right)$ and $\left(\frac{2}{3}, \frac{1}{3}, -\frac{2}{3}\right)$.

29. Find the first four Taylor polynomials for $\ln x$ about $x = 2$.

30. Test the convergence of the following series

(a) $\sum_{k=1}^{\infty} \frac{k^k}{k!}$

(b) $\sum_{k=1}^{\infty} \frac{1}{2^k - 1}$

31. Show that $|x|$ is continuous everywhere.

(6 × 4 = 24 Marks)

SECTION – D

Answer any **two** questions. These question carries **15** marks.

32. (a) Sketch a graph of $y = \frac{x^2 - 1}{x^3}$ and identify the locations of all asymptotes, intercepts, relative extrema, and inflection points.

(b) Find the slope of circle $x^2 + y^2 = 25$ at the point (3, –4).

33. (a) Use implicit differentiation to find $\frac{dy}{dx}$ if $y^2 = x^2 + \sin xy$.

(b) Find the tangent to the curve $x^3 + y^3 - 9xy = 0$ at the point (2,4).

34. (a) Find the local extreme values of the function $f(x, y) = xy - x^2 - y^2 - 2x - 2y + 4$.

(b) At what point or points on the circle $x^2 + y^2 = 1$ does $f(x, y) = xy$ have an absolute maximum, and what is that maximum?

35. (a) Find the interval of convergence and radius of convergence of $\sum_{k=1}^{\infty} \frac{(x-5)^k}{k^2}$. (6)

(b) Find the values of x for which the power series $\sum_{k=1}^{\infty} k!\, x^k$ converge. (3)

(c) Find the values of x for which the power series $\sum_{k=1}^{\infty} \frac{(-1)^{k-1} x^{2k-1}}{2k-1}$ converge.

(2 × 15 = 30 Marks)
